GALAXIES AND COSMOLOGY

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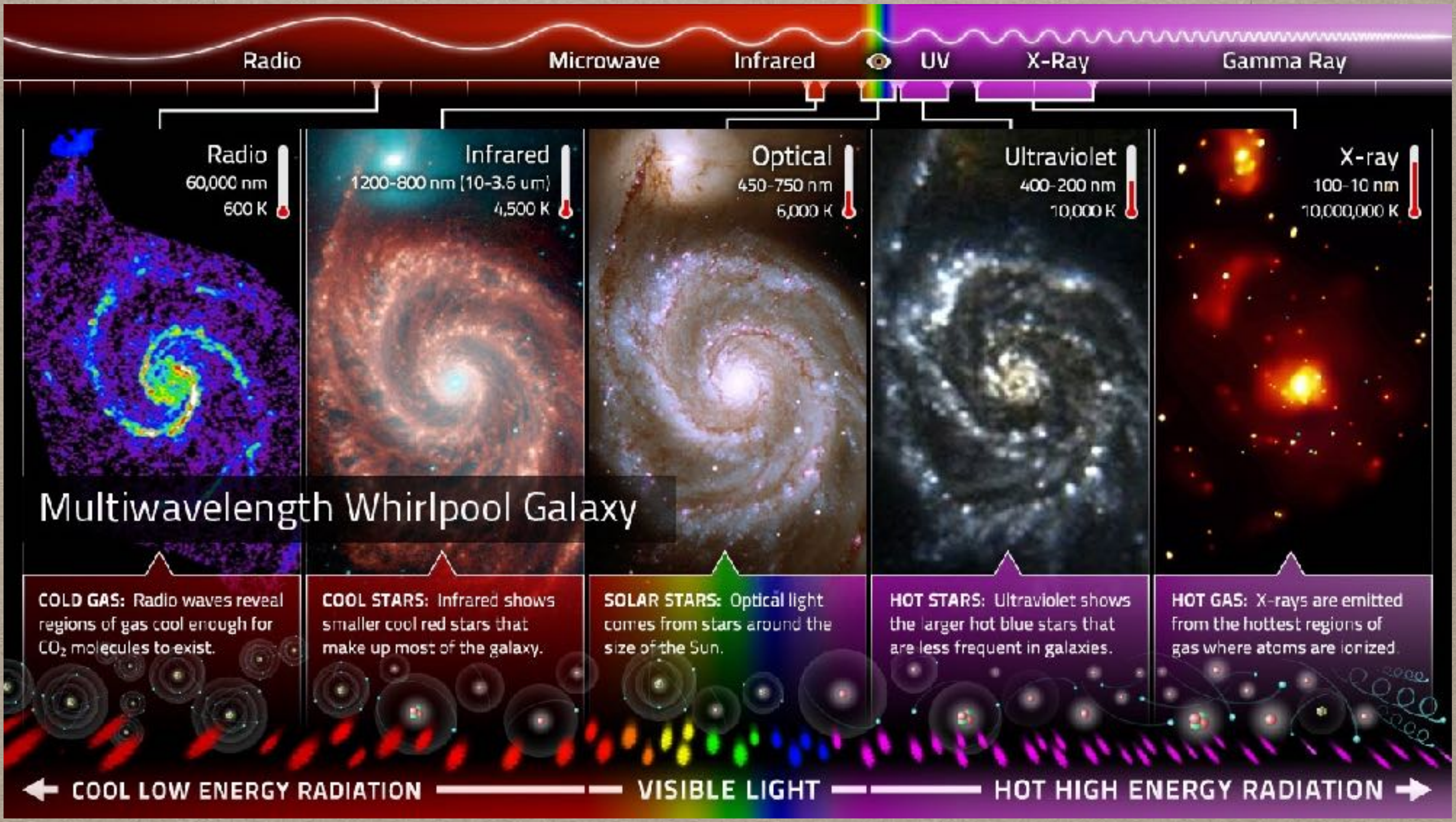
WHY GALAXIES?

- Galaxies are useful tracers of the underlying density field at large-scales relevant for cosmology.
- Understanding the galaxy formation and evolution is important for understanding radiation and thermal history of the universe.
- Imaging and spectroscopic observations are possible. We would like to derive, (i) size & shape, (ii) star formation rate, (iii) metallicity, (iv) age, (v) stellar mass, (vi) dark matter mass, (vii) luminosity functions, (viii) spatial clustering and (xv) redshift evolution of all the quantities.

GALAXIES: MULTIWAVELENGTH



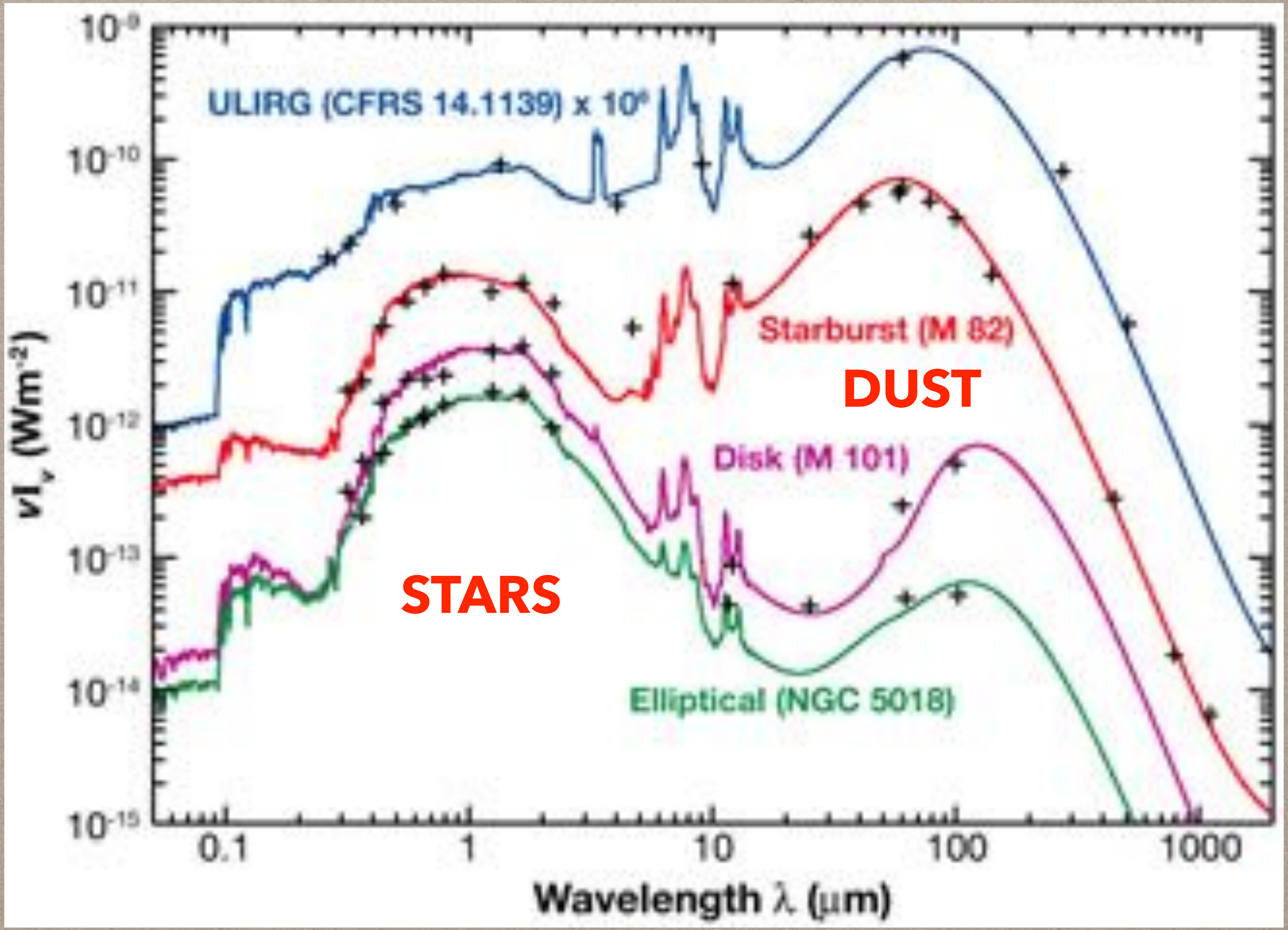
GALAXIES: MULTIWAVELENGTH

Band	Energy	Dominant Physical Processes	Principal Sources	Other Important Contributors	Physical Conditions Traced
X-ray	0.1–100 keV	Thermal bremsstrahlung (free-free emission); collisionally excited line emission; inverse Compton scattering; accretion-powered emission	Hot diffuse gas, X-ray binaries, AGN, supernova remnants	Cataclysmic variables, coronally active binaries, supersoft X-ray sources, hot white dwarfs	Hot gas (10^6–10^8 K), shocks, stellar remnants, accretion, energetic feedback
UV	10–400 nm	Photospheric emission from hot stars; nebular continuum; resonance-line emission; thermal emission from accretion disks	O and B stars, H II regions, AGN accretion disks	White dwarfs, post-AGB stars, extreme horizontal branch stars, blue stragglers	Recent star formation, evolved stellar populations, ionized gas, AGN activity
Optical	400–700 nm	Stellar continuum emission; hydrogen recombination lines; forbidden-line emission; AGN continuum emission	Main-sequence stars, giant stars, H II regions, AGN emission-line regions	Planetary nebulae, supernovae, Wolf–Rayet stars, novae	Stellar mass, stellar ages, metallicity, ionization state, star formation

GALAXIES: MULTIWAVELENGTH

Band	Approx. Range	Dominant Physical Processes	Principal Sources	Other Important Contributors	Physical Conditions Traced
Near-Infrared (NIR)	0.7–5 µm	Stellar photospheric emission; thermal emission from warm dust; molecular rovibrational transitions	Red giants, red supergiants, old stellar populations, embedded young stars	AGB stars, TP-AGB stars, AGN-heated dust, brown dwarfs	Stellar mass, evolved stellar populations, obscured star formation
Mid-Infrared (MIR)	5–30 µm	Thermal emission from warm dust; PAH emission; fine-structure ionic lines; reprocessed UV/optical radiation	Star-forming regions, photodissociation regions (PDRs), AGN dusty torus	Circumstellar dust around AGB stars, shocked dust, planetary nebulae	Dust heating, obscured star formation, AGN activity, ISM
Far-Infrared (FIR)	30–300 µm	Thermal emission from cold dust grains in approximate equilibrium with the radiation field	Molecular clouds, diffuse ISM dust, star-forming galaxies	Dust heated by old stellar populations ("cirrus"), AGN-heated dust	Dust mass, obscured star formation rate, radiation field intensity
Submillimeter (Sub-mm)	0.3–1 mm	Thermal emission from very cold dust; molecular rotational line emission (especially CO)	Giant molecular clouds, dense molecular gas, dusty high-redshift galaxies	Cold circumgalactic dust, dense star-forming cores	Molecular gas reservoirs, cold dust content
Millimeter (mm)	1–10 mm	Molecular rotational transitions; thermal dust continuum; free-free emission	Molecular gas (CO, HCN, HCO^+, CS), dense cores, star-forming clouds	Sunyaev–Zel'dovich effect, AGN jets, cold dust continuum	Molecular gas mass, gas kinematics, dense gas fraction

SPECTRAL ENERGY DISTRIBUTION



GENERATING GALAXY SPECTRA ENERGY DISTRIBUTION:

$$F_{obs}(\lambda, t) = \int_0^t d\tau\, f_{\mathrm{SSP}}(\lambda, \tau, Z) SFR(t, \tau) T(\lambda, t, \tau)$$

SFR: Single burst, continuous (i.e independent of time), exponentially decreasing/increasing, recurring etc.,

T: Transmission effects - dust scattering/emission, gas absorption and nebular emission.

f_{SSP}: Simple Stellar Populations born at the same stage.

INITIAL MASS FUNCTION (IMF)

- Salpeter IMF

$$\xi(M) = 0.03(\frac{M}{M_\odot})^{-1.35}$$

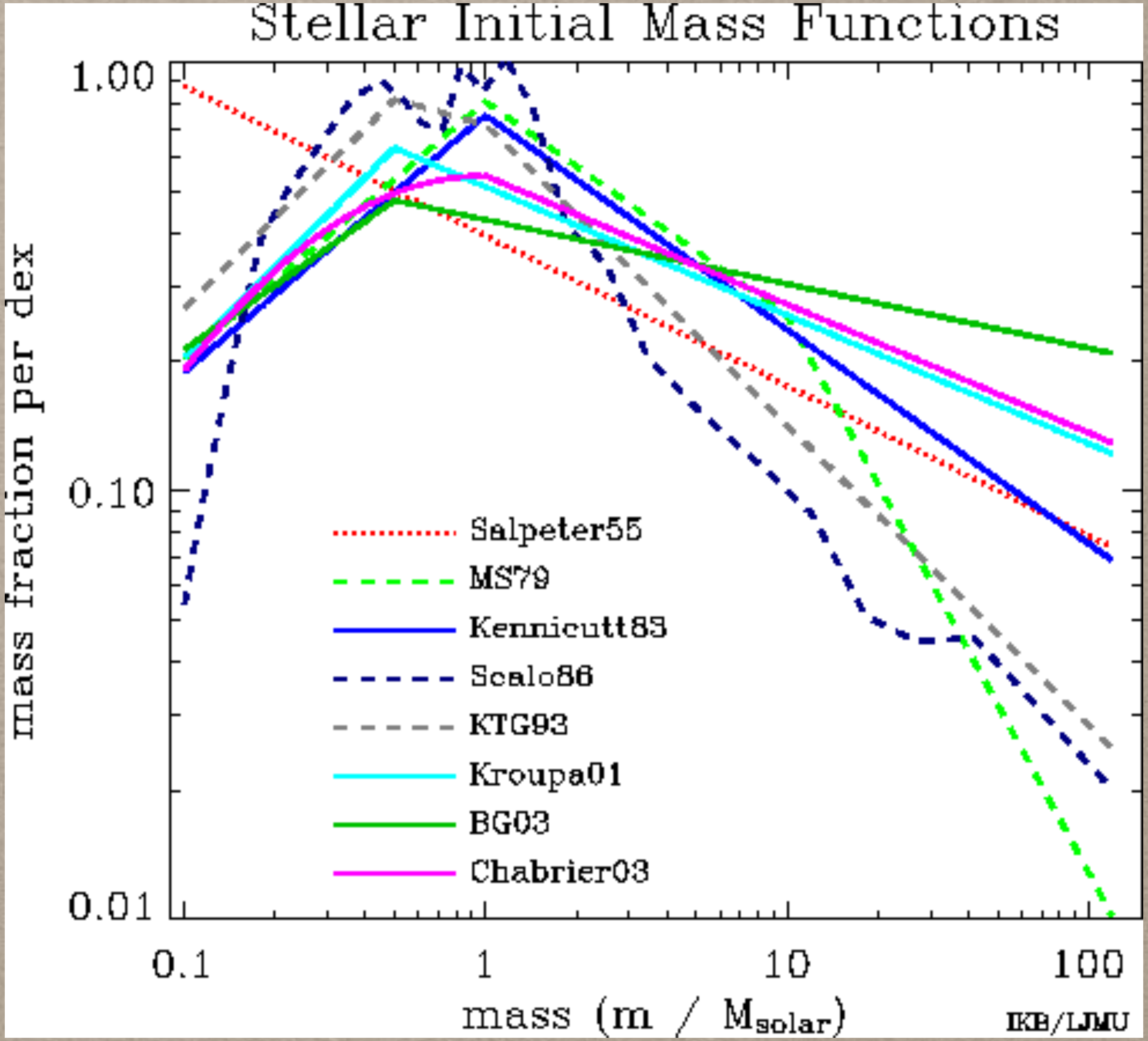
- Scalo IMF

- Chabrier IMF

TABLE 1
DISK IMF AND PDMF FOR SINGLE OBJECTS

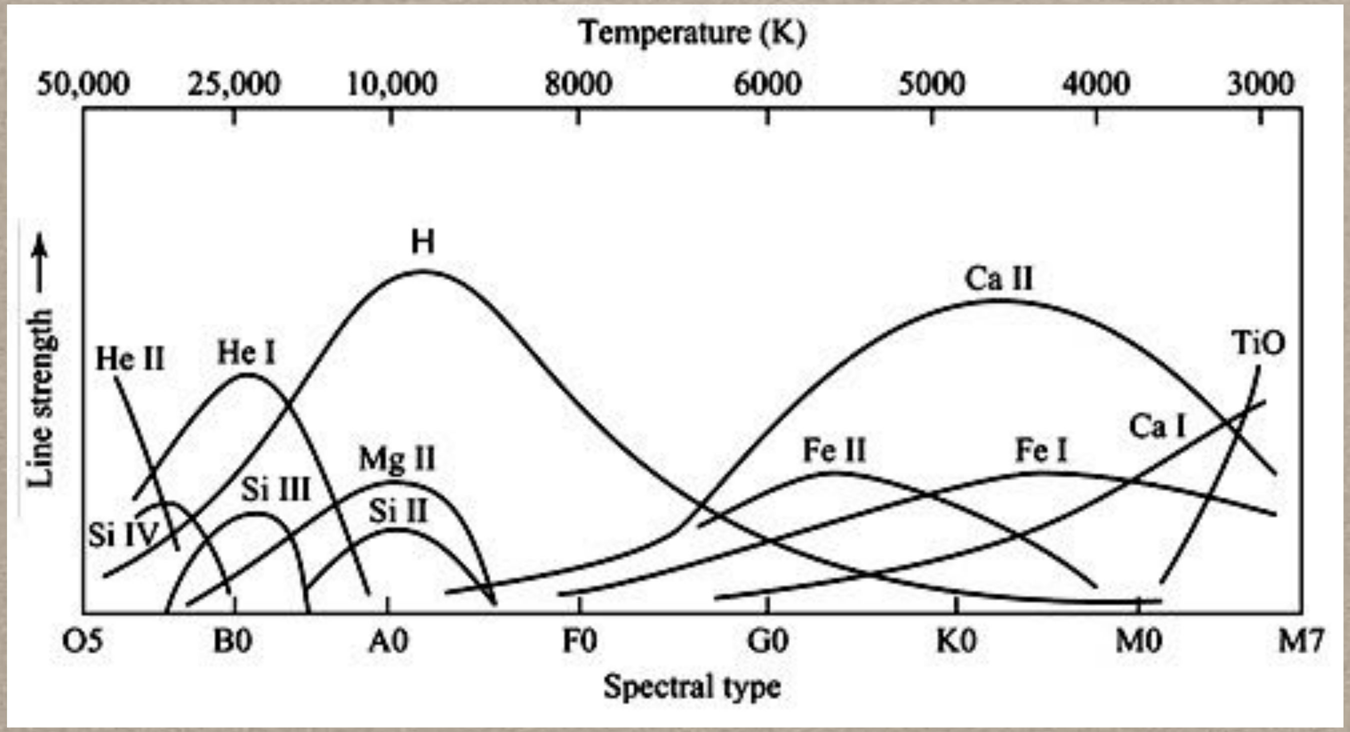
Parameter	IMF	PDMF
$m \le 1.0\ M_\odot$, $\xi(\log m) = A \exp[-(\log m - \log m_c)^2/2\sigma^2]$		
A	$0.158^{+0.051}_{-0.046}$	$0.158^{+0.051}_{-0.046}$
m_c	$0.079^{-0.016}_{+0.021}$	$0.079^{-0.016}_{+0.021}$
σ	$0.69^{-0.01}_{+0.05}$	$0.69^{-0.01}_{+0.05}$
$m > 1.0\ M_\odot$, $\xi(\log m) = A m^{-x}$		
A	4.43×10^{-2}	
x	1.3 ± 0.3	
$0 \le \log m \le 0.54$:		
A		4.4×10^{-2}
x		4.37
$0.54 \le \log m \le 1.26$:		
A		1.5×10^{-2}
x		3.53
$1.26 \le \log m \le 1.80$:		
A		2.5×10^{-4}
x		2.11

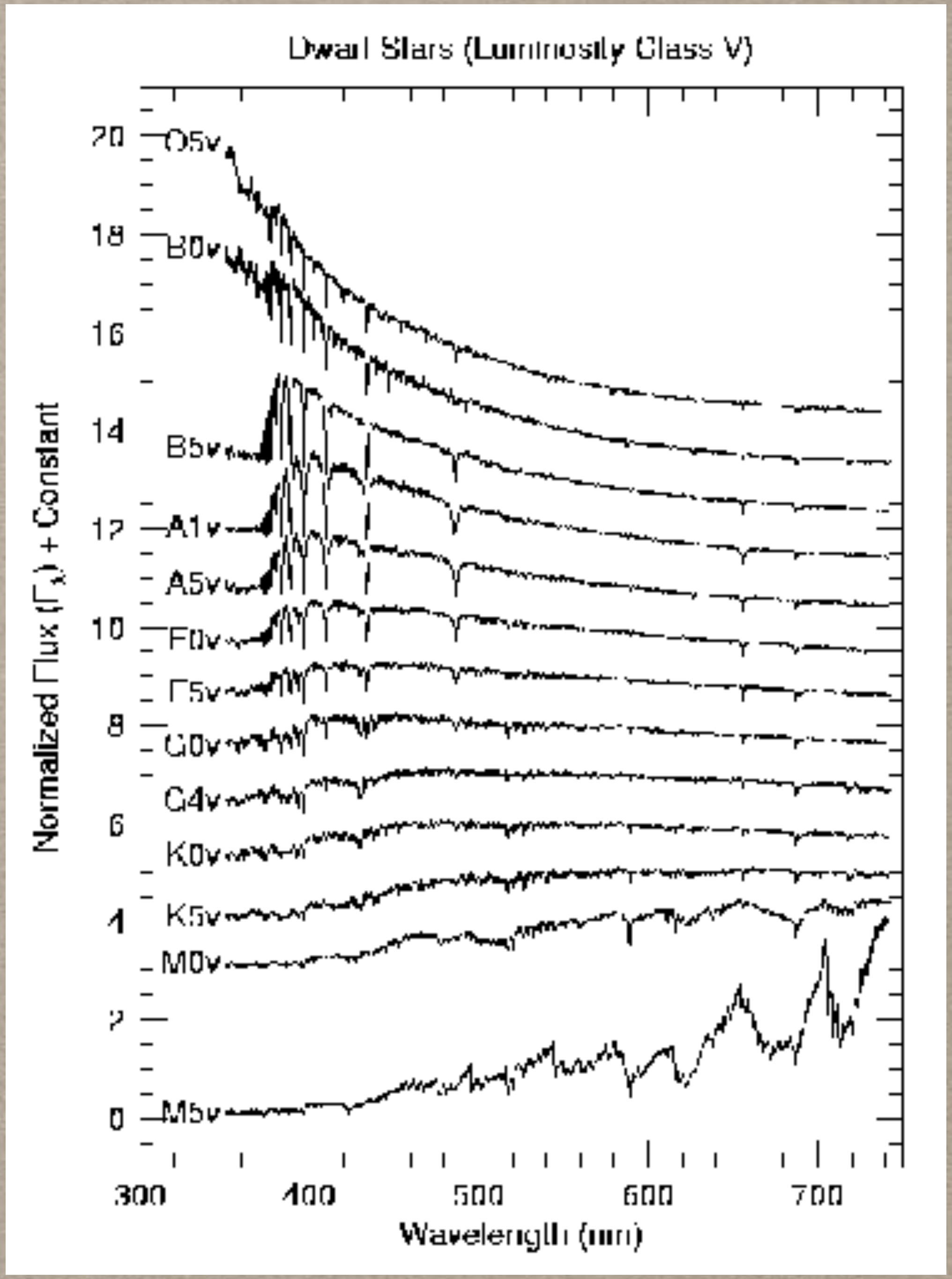
NOTE.—For unresolved binary systems, the coefficients are given by eq. (18). The normalization coefficient A is in $(\log M_\odot)^{-1}$ pc^{-3}.



STAR SPECTRA

- Collisional ionisation and excitation equilibrium make spectrum deviate from BB.

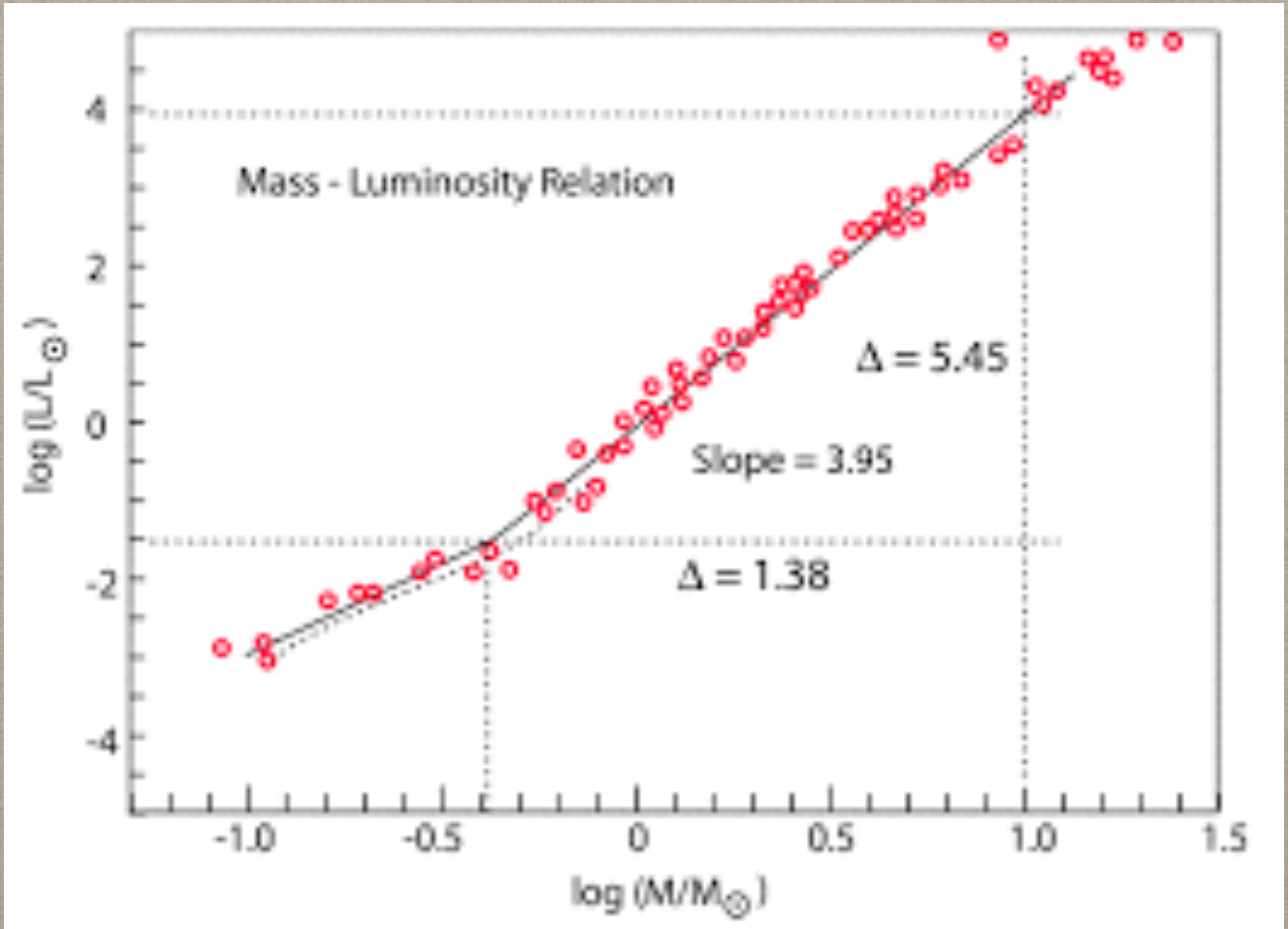




$$\frac{N_2}{N_1} = \frac{g_2}{g_1} exp(-\Delta E/KT_{ex})$$

$$\frac{n(X^{(r+1)})n_e}{n(X^{(r)})} = 2\ \frac{g_{r+1,1}}{g_{r,1}} \left(\frac{2\pi m_e KT}{h^2}\right)^{3/2} exp(-\phi/KT)$$

MASS,LUMINOSITY & LIFETIME



BASIC PROPERTIES OF STARS

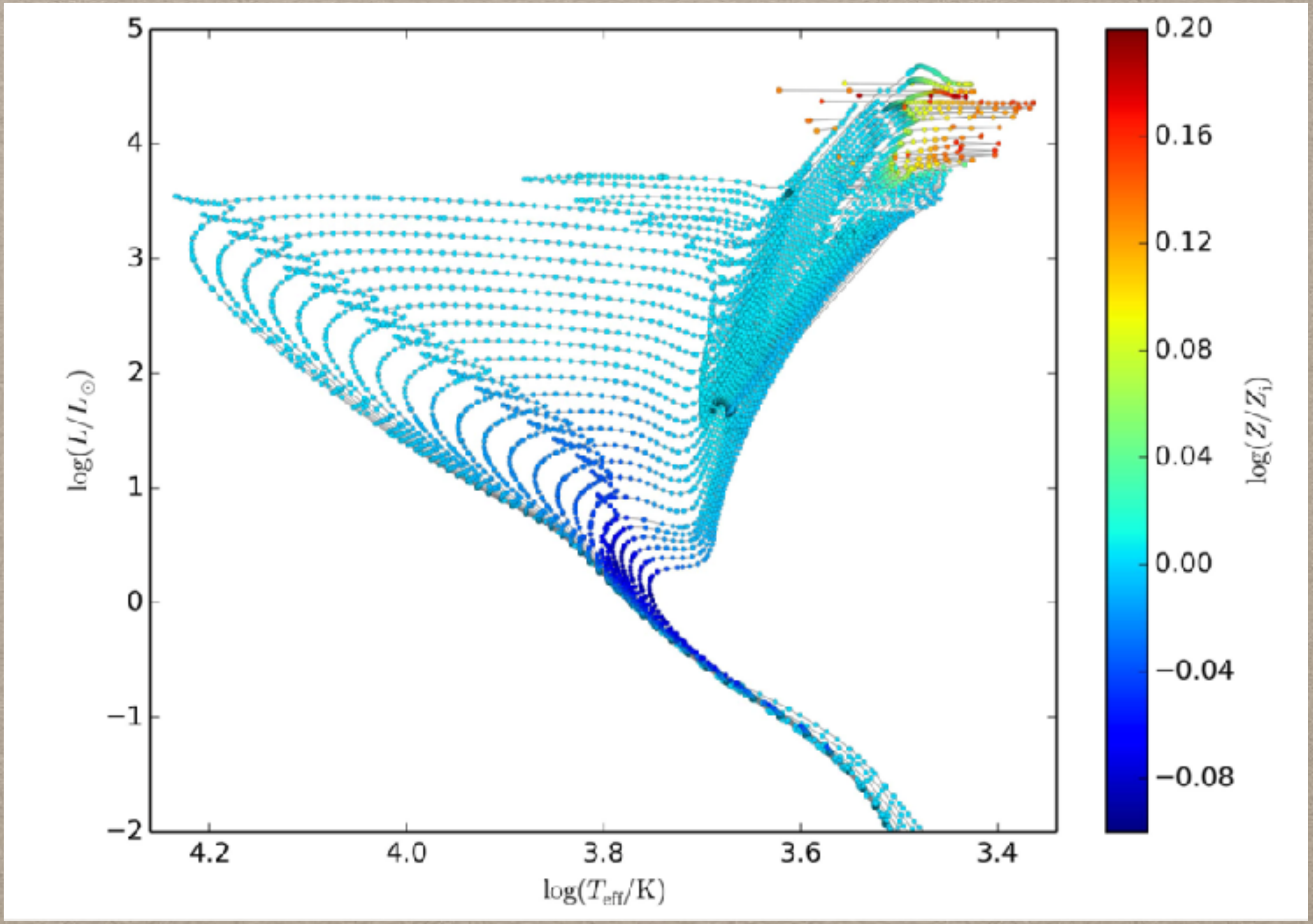
Table 7.1. *Representative stellar data*

$M_{init.}$ ($M_\odot$)	On zero-age main sequence (ZAMS): log $L/L_\odot$	T_{eff} (10^3K)	Sp	M_V	$B-V$	τ_{ms} Myr or Gyr[a]	τ_{tot} Myr or Gyr[b]
120	6.2	53(59)				3.0(2.9)	3.9(3.2)
60	5.7	48(52)				3.5(3.9)	4.0(4.3)
40	5.4	44(48)	O5	−5.6	−0.32	4.4(5.1)	5.0(5.5)
20	4.6	35(39)	O8	−5.0	−0.31	8.2(9.4)	9.0(10.2)
12	4.0	28(32)	B0.5	−4.0	−0.28	16(18)	18(20)
7	3.3	21(25)	B2	−1.5	−0.22	43(45)	48(50)
5	2.8	17(21)	B4	−0.8	−0.19	94(88)	107(100)
3	1.9(2.1)	12.2(16.1)	B7	−0.2	−0.12	350(290)	440(340)
2	1.2(1.4)	9.1(12.2)	A2	1.4	0.05	1.16(.86)	1.36(1.03)
1.5	.68(.92)	7.1(9.6)	F3	3.0	0.40	2.7(1.84)	(2.0)
1.0	−.16(.15)	5.64(6.71)	G5	5.2	0.65	10.0(7.3)	
0.9	−.39(−.07)	5.30(6.31)	K0	5.9	0.89	15.5(10.7)	
0.8	−.61(−.31)	4.86(5.86)	K2	6.4	0.94	25(15)	

[a] Time to end of core H burning.
[b] Time to end of C or He burning.
Sources: Schaller *et al.* (1992); Meynet *et al.* (1994); Tinsley (1980); Allen (1973).

see: Pagel (1993) *Instituto Astrofisica di Canarias Winter School*

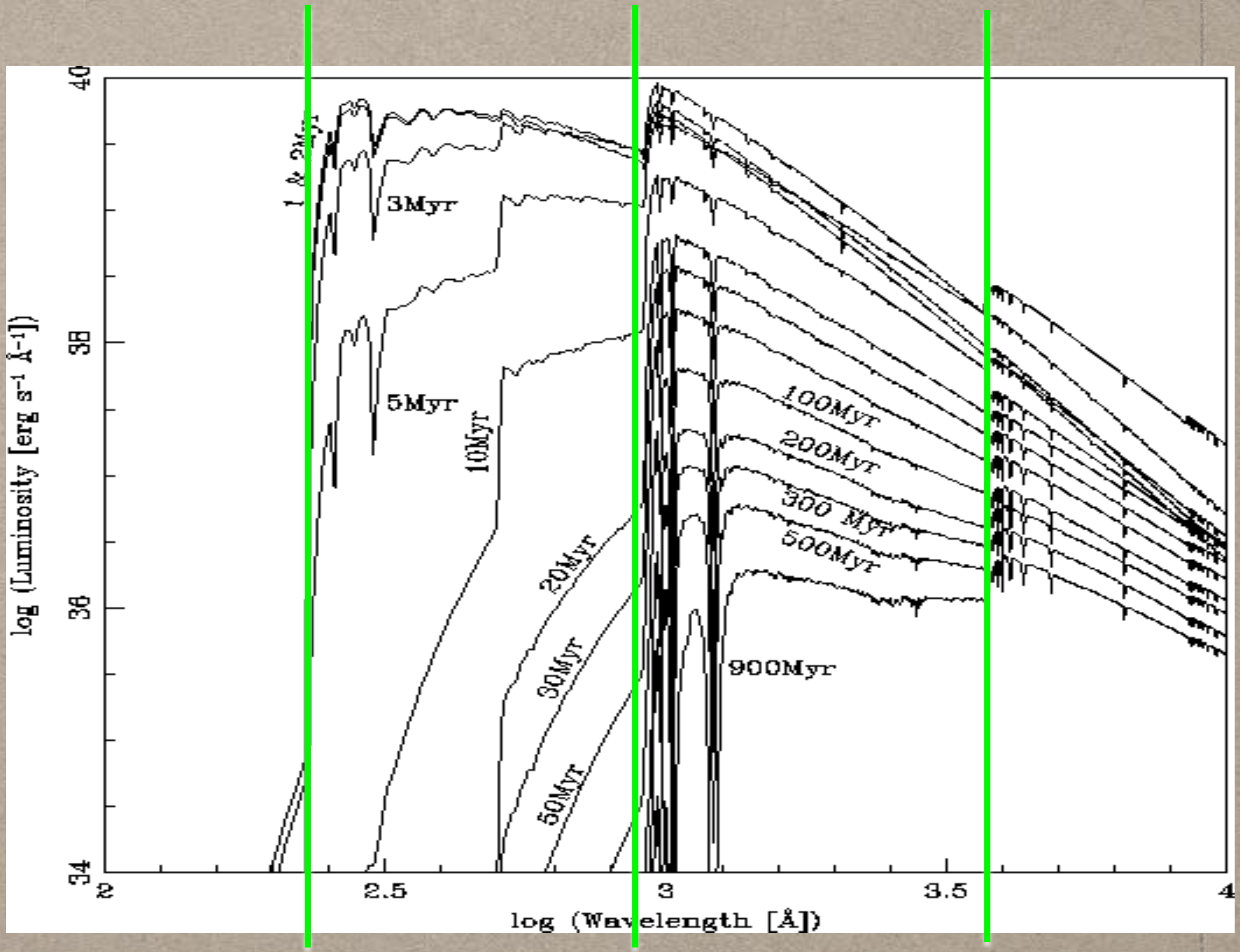
STELLAR ISOCHRONE



Marigo, 2017, ApJ, 835,77

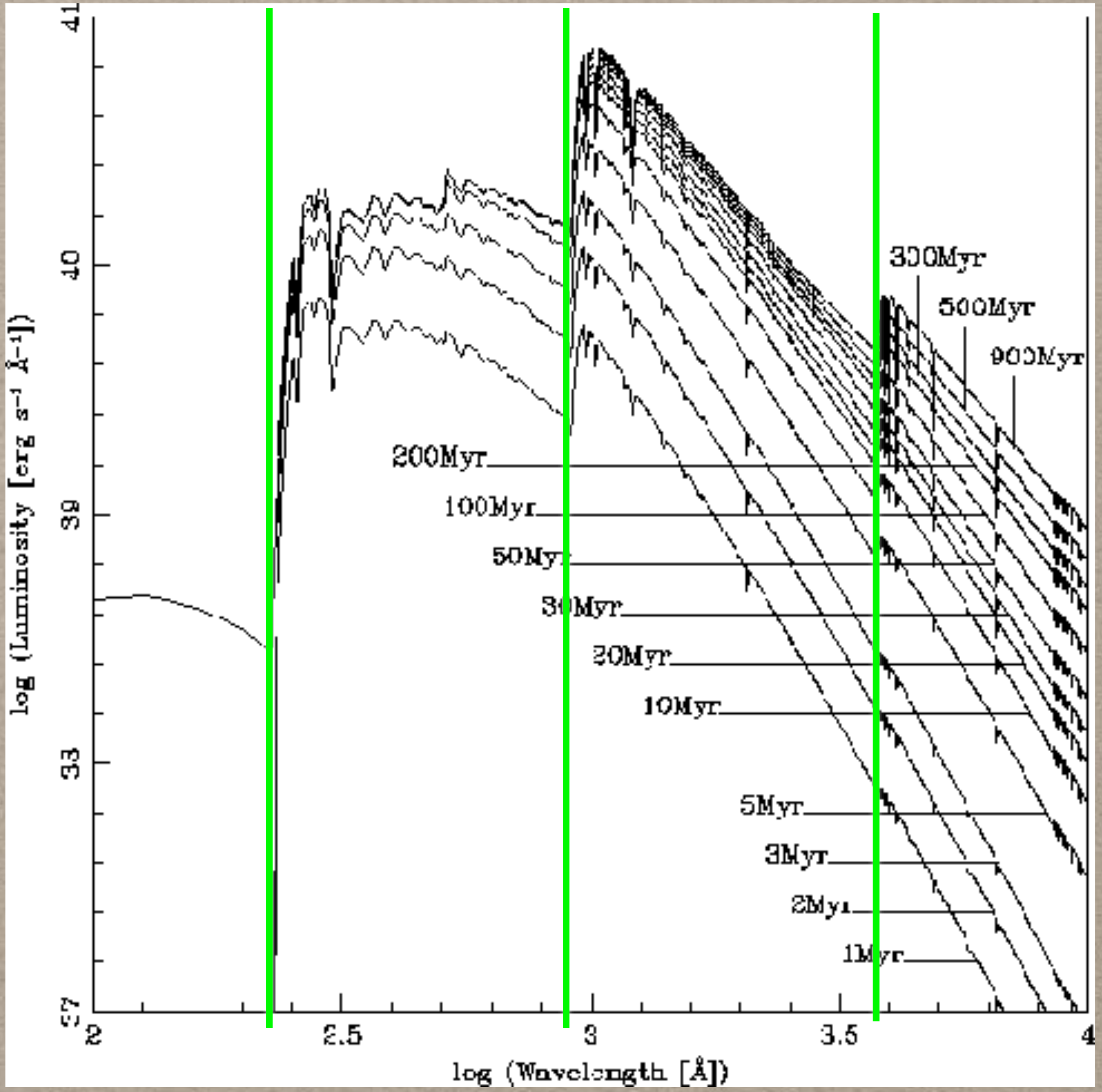
SSP EVOLUTION

- Salpeter IMF.
- 10^6 $M_\odot$ single burst.
- $Z = 1/20\ Z_\odot$.
- "STARBURST99"
- Metallicity, Binary fraction, IMF, mass cut-offs will affect the computed spectrum.

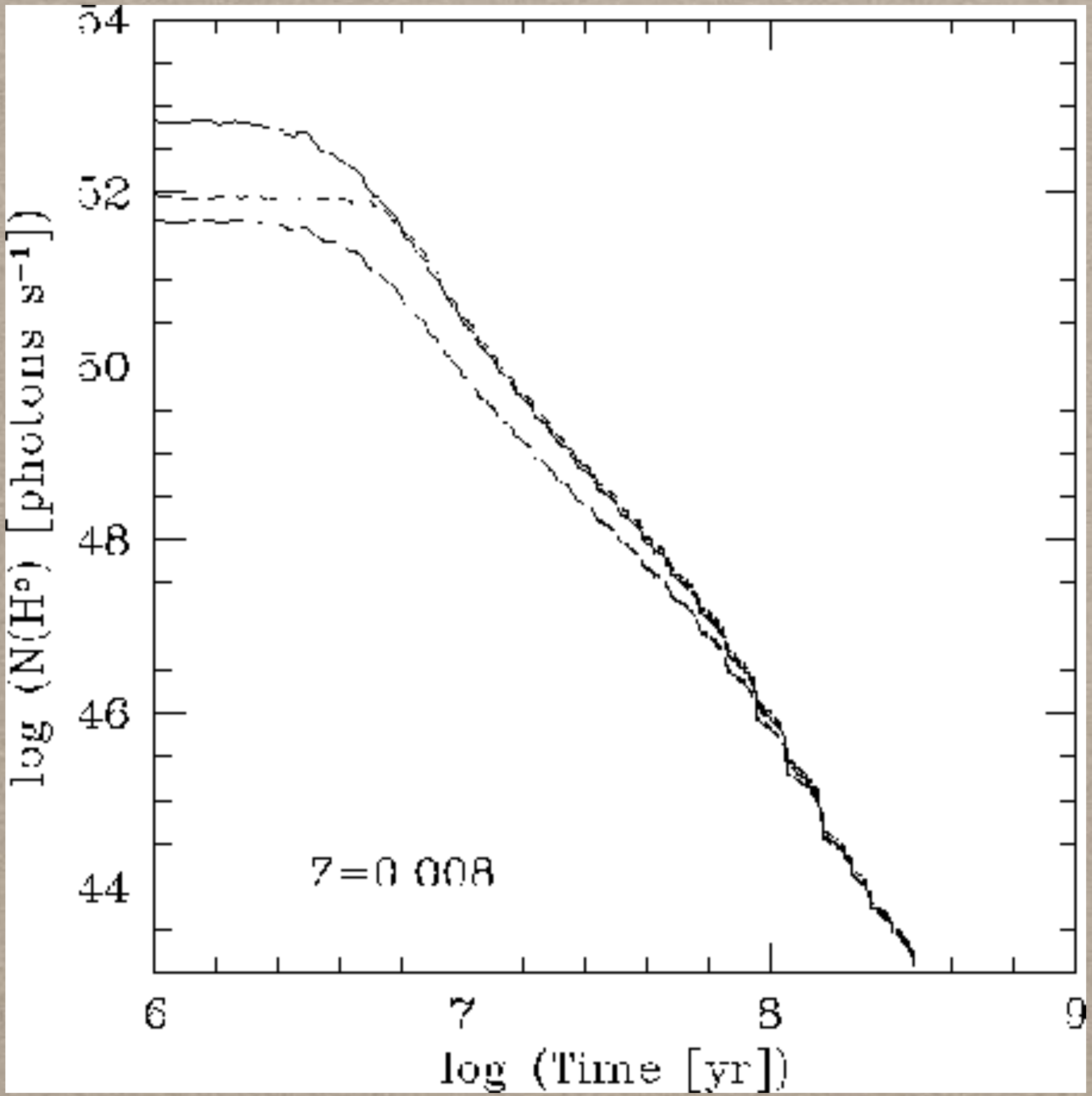


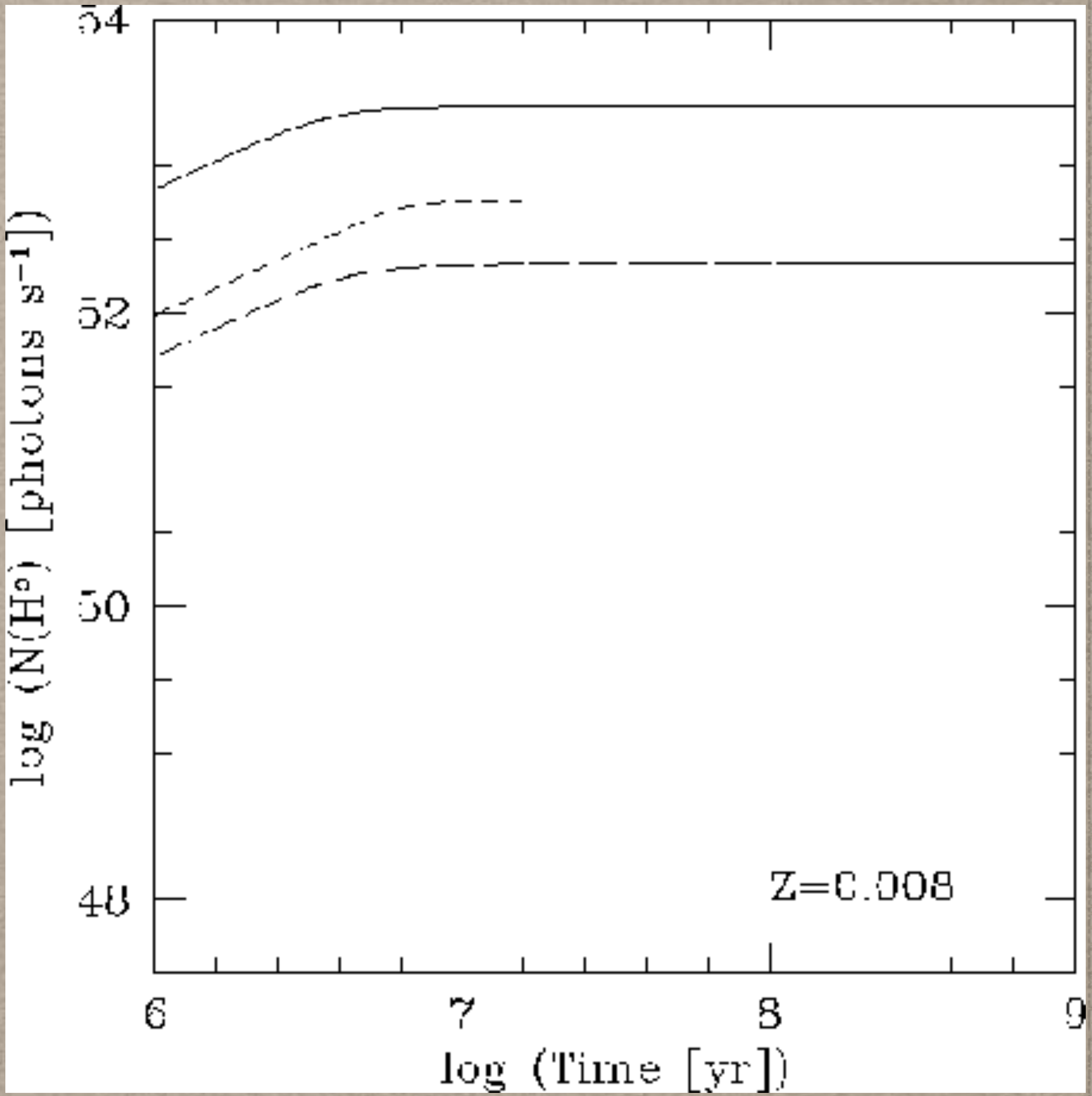
CONTINUOUS STARFORMATION

- Salpeter IMF.
- $1 M_{\odot}\ yr^{-1}$ continuous.
- $Z = 1/20\ Z_{\odot}$.
- "STARBURST99"
- Without ISM absorption.



CONTINUOUS STARFORMATION





$M = 10^6\ M_\odot$; M_low = 1

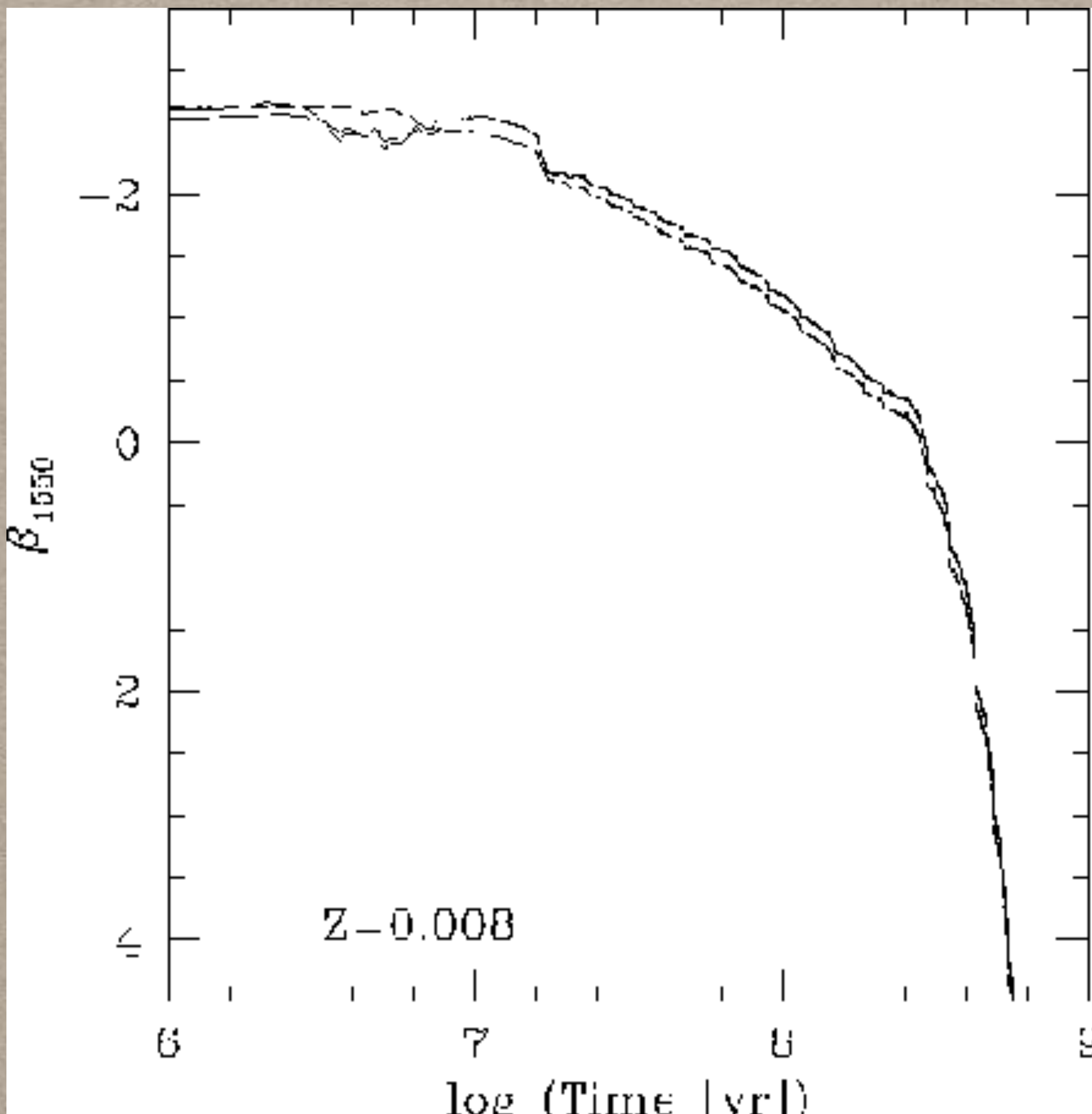
solid: M_up = 100 $M_\odot$, $\alpha = 2.35$

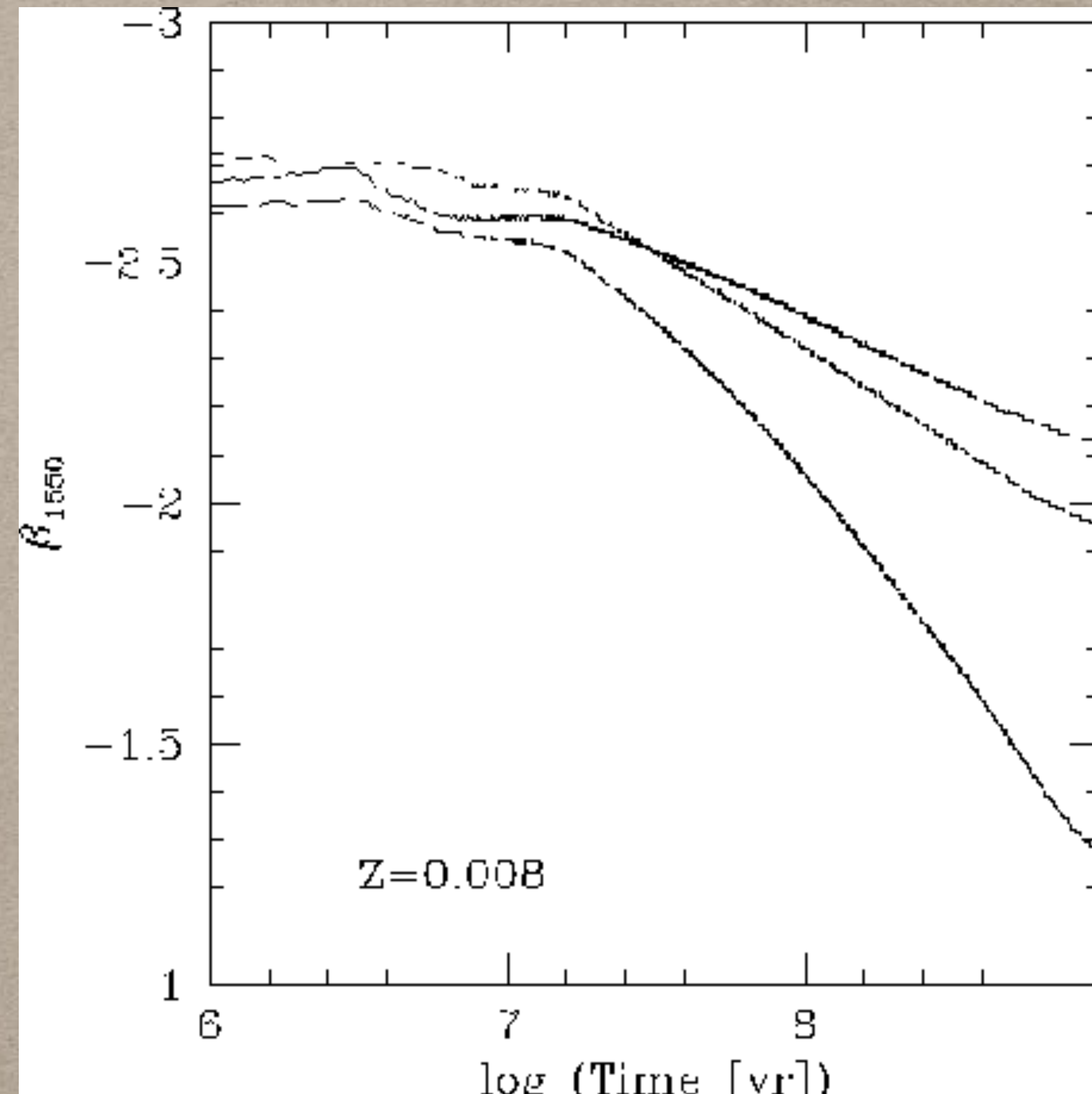
dashed: M_up = 100 $M_\odot$, $\alpha = 3.30$

long-dashed: M_up = 30 $M_\odot$, $\alpha = 2.35$

SFR = 1 $M_\odot\ yr^{-1}$

CONTINUOUS STARFORMATION





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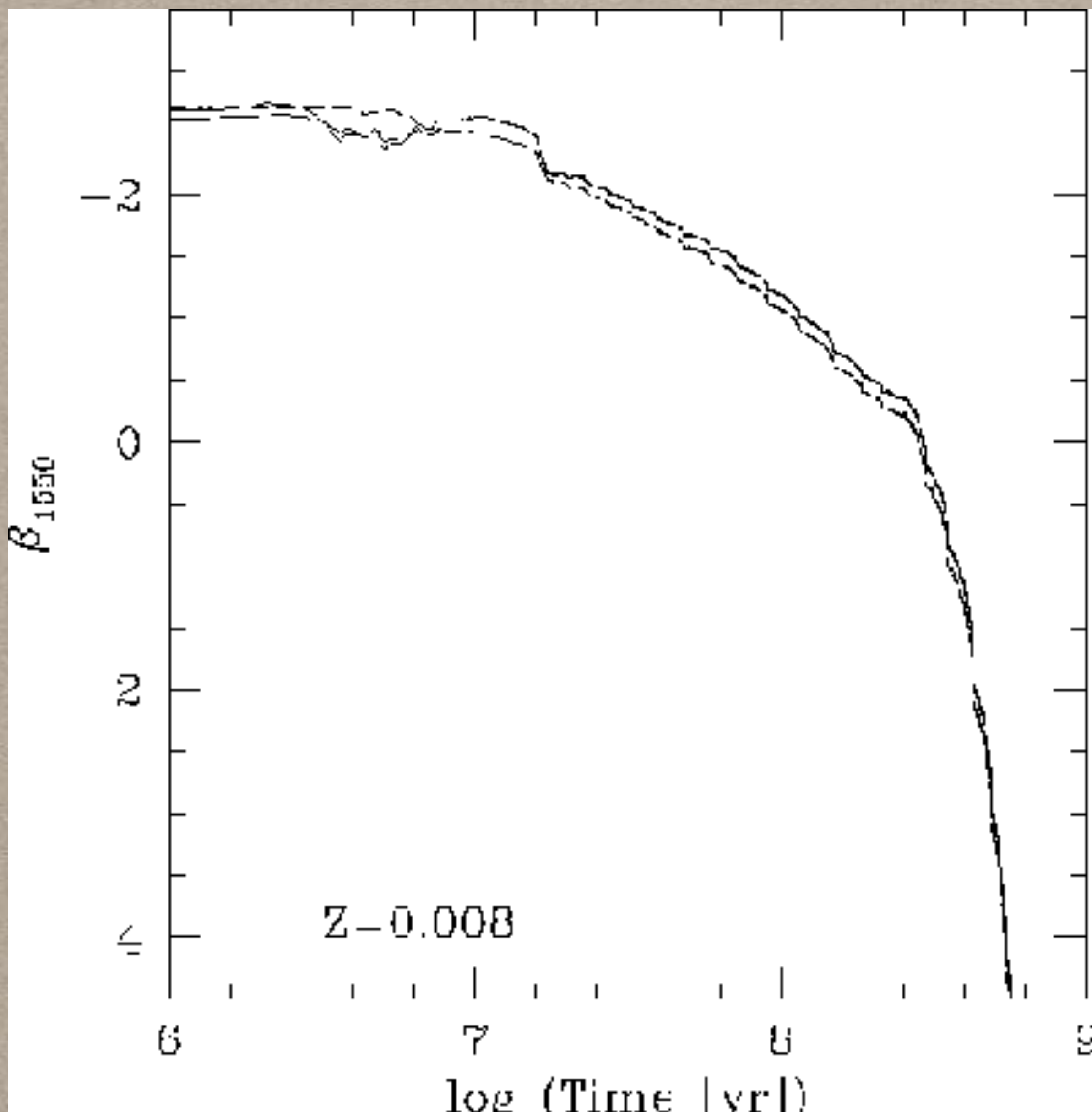
solid: M_up = 100 $M_\odot$, $\alpha = 2.35$

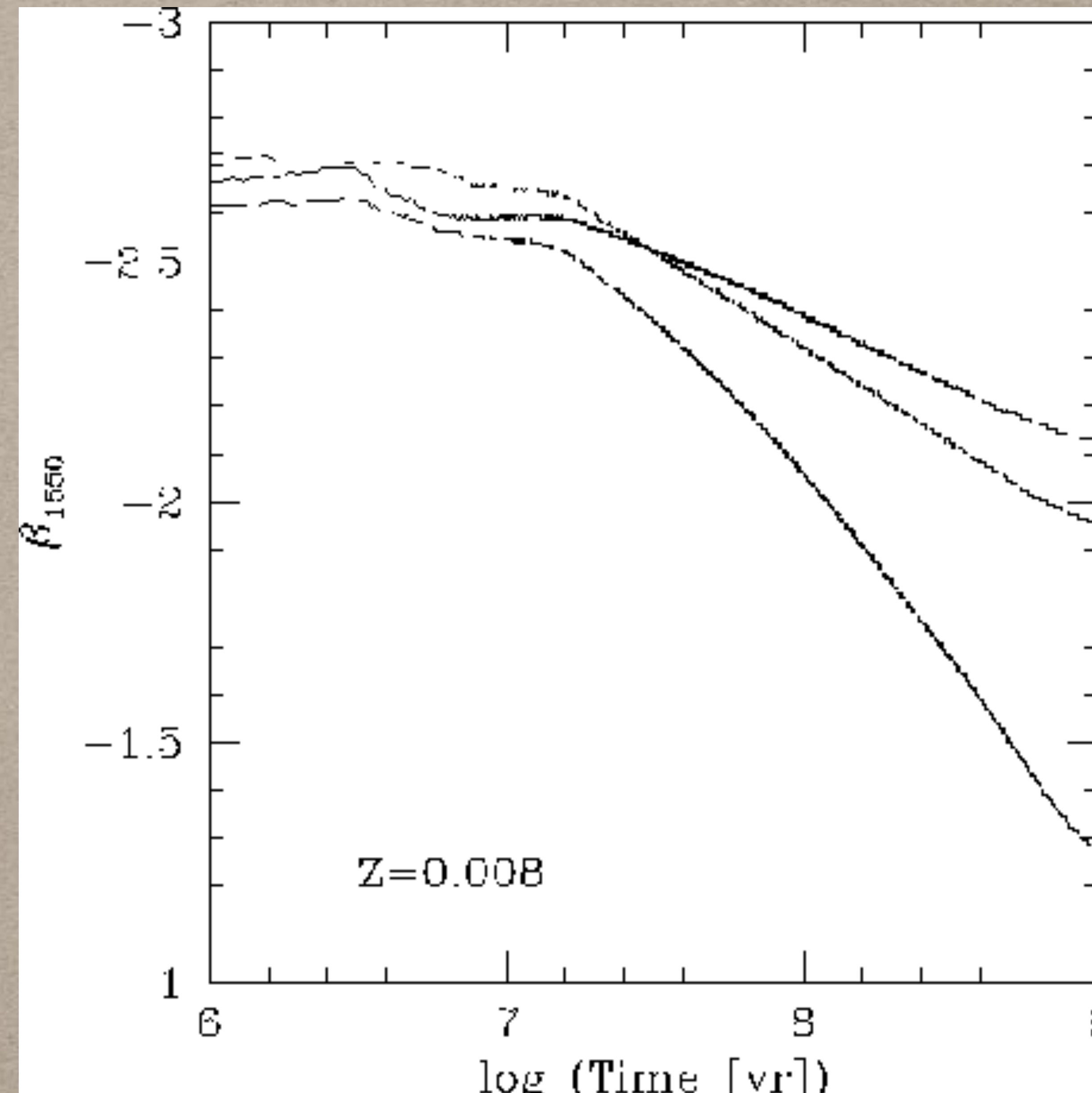
dashed: M_up = 100 $M_\odot$, $\alpha = 3.30$

long-dashed: M_up = 30 $M_\odot$, $\alpha = 2.35$

SFR = 1 $M_\odot\ yr^{-1}$

CONTINUOUS STARFORMATION





$M = 10^6\ M_\odot$; M_low = 1

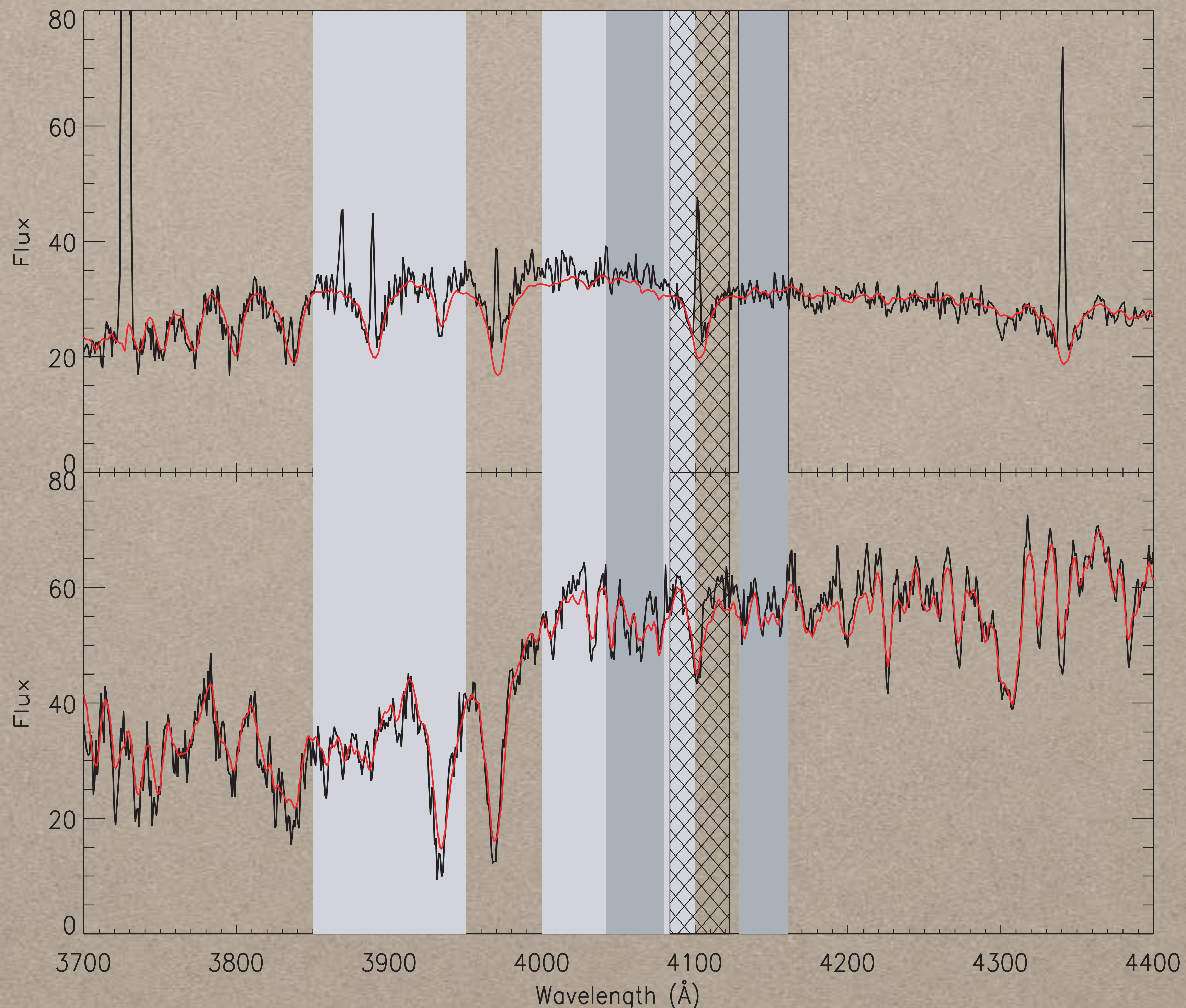
solid: M_up = 100 $M_\odot$, $\alpha = 2.35$

dashed: M_up = 100 $M_\odot$, $\alpha = 3.30$

long-dashed: M_up = 30 $M_\odot$, $\alpha = 2.35$

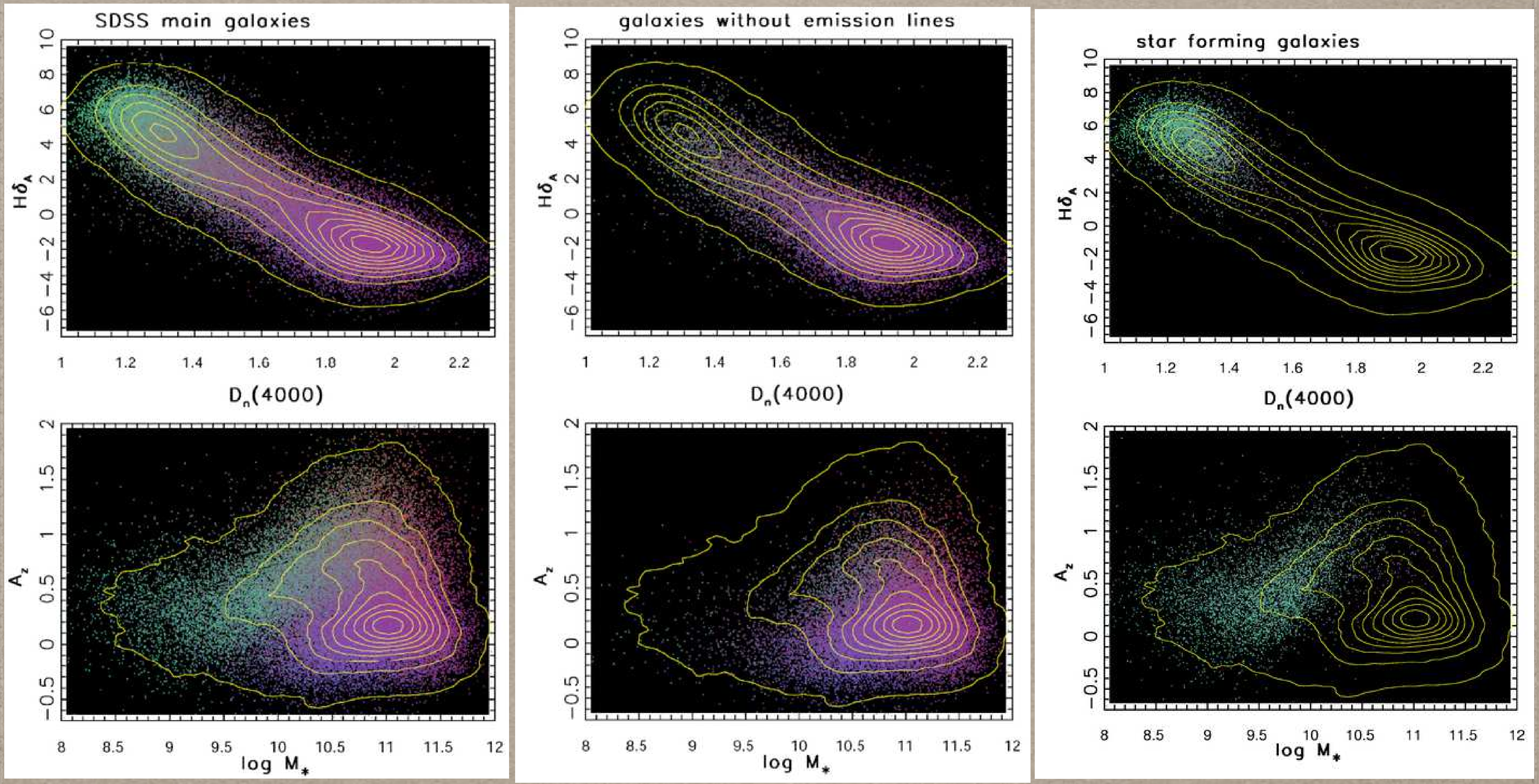
SFR = 1 $M_\odot\,yr^{-1}$

D_N(4000) AND GALAXY PARAMETER

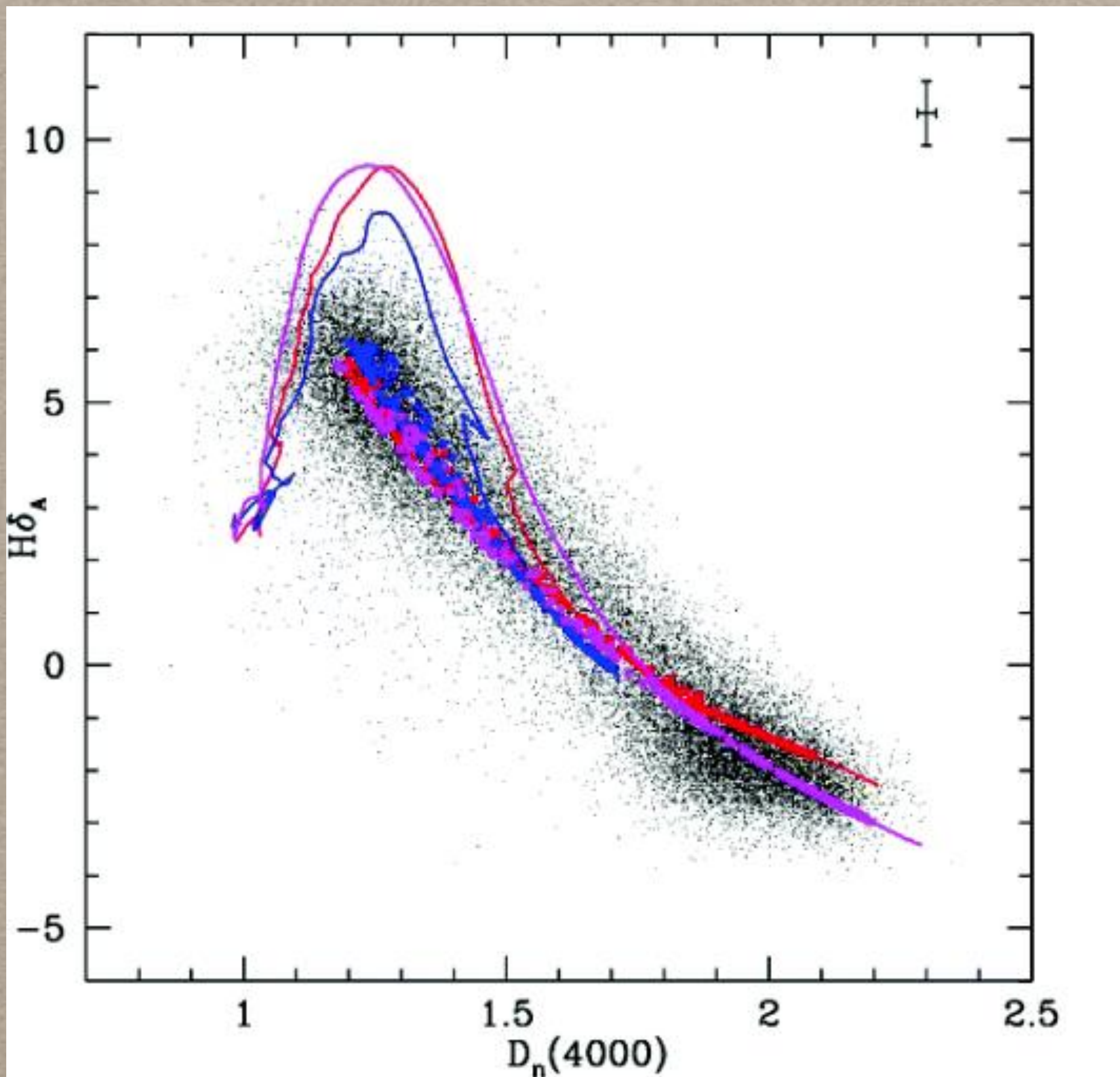


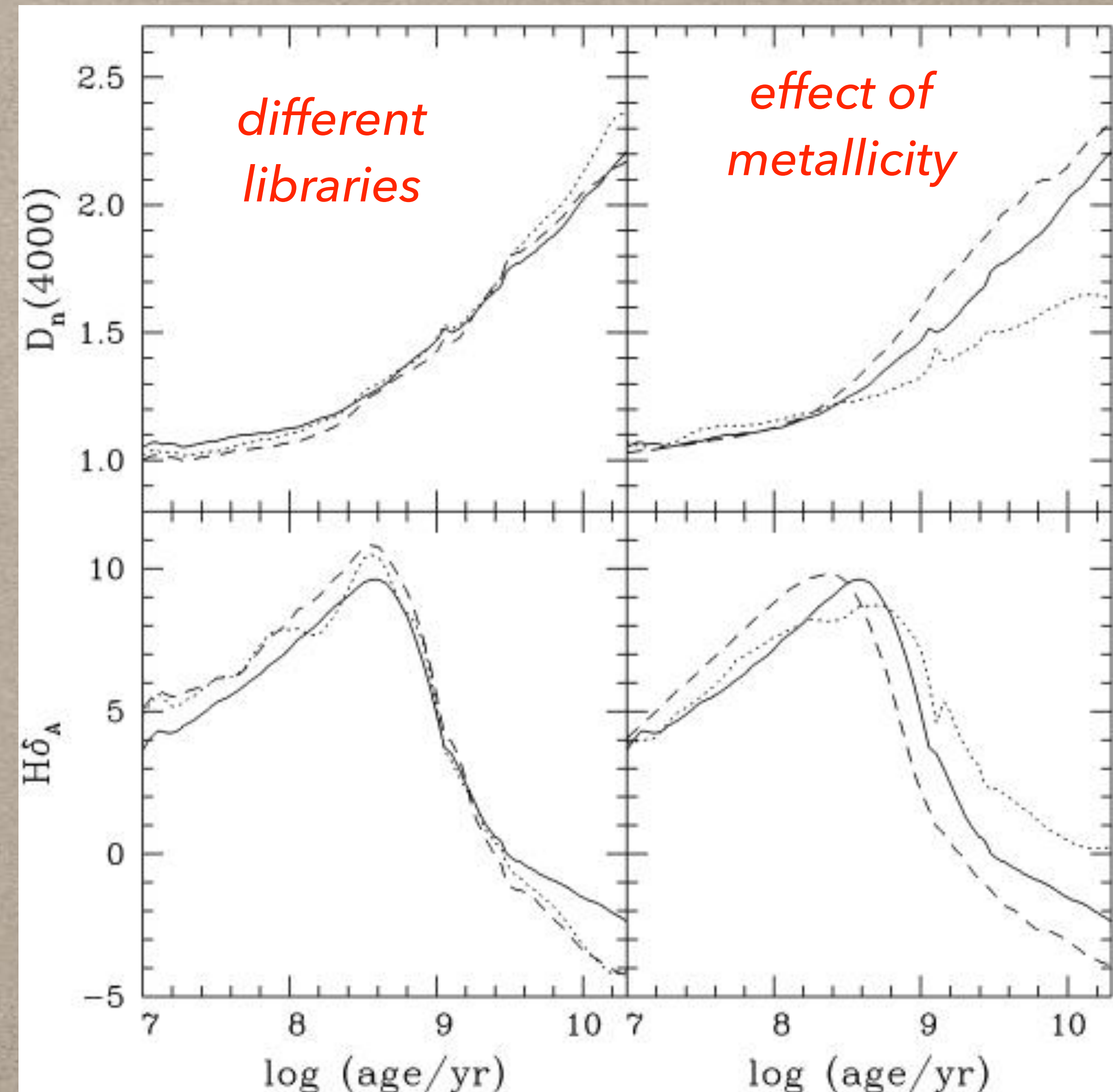
Kauffmann et al 2003, MNRAS, 341, 33

D_N(4000) AND GALAXY PARAMETER



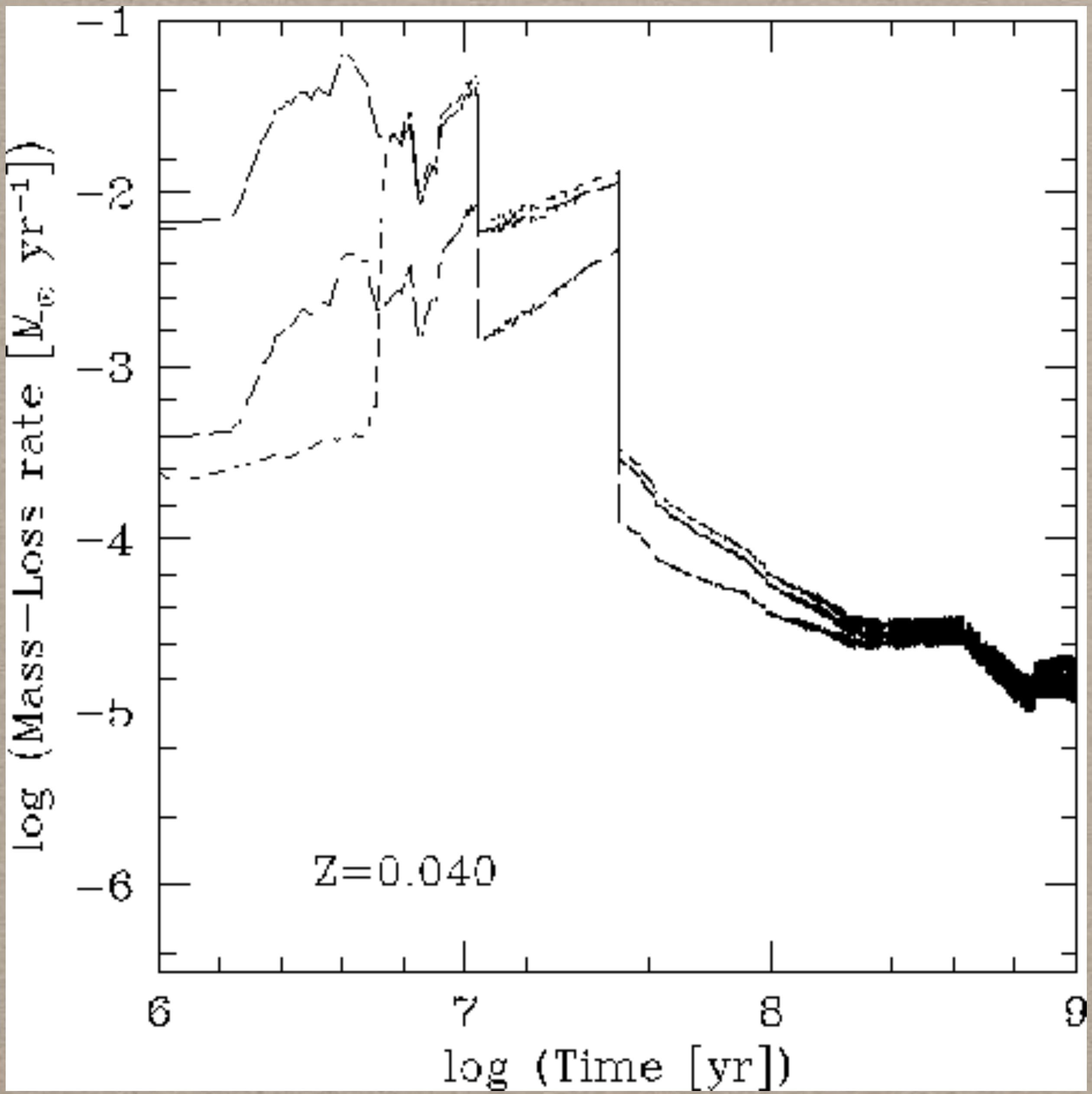
D_N(4000) AND GALAXY PARAMETER

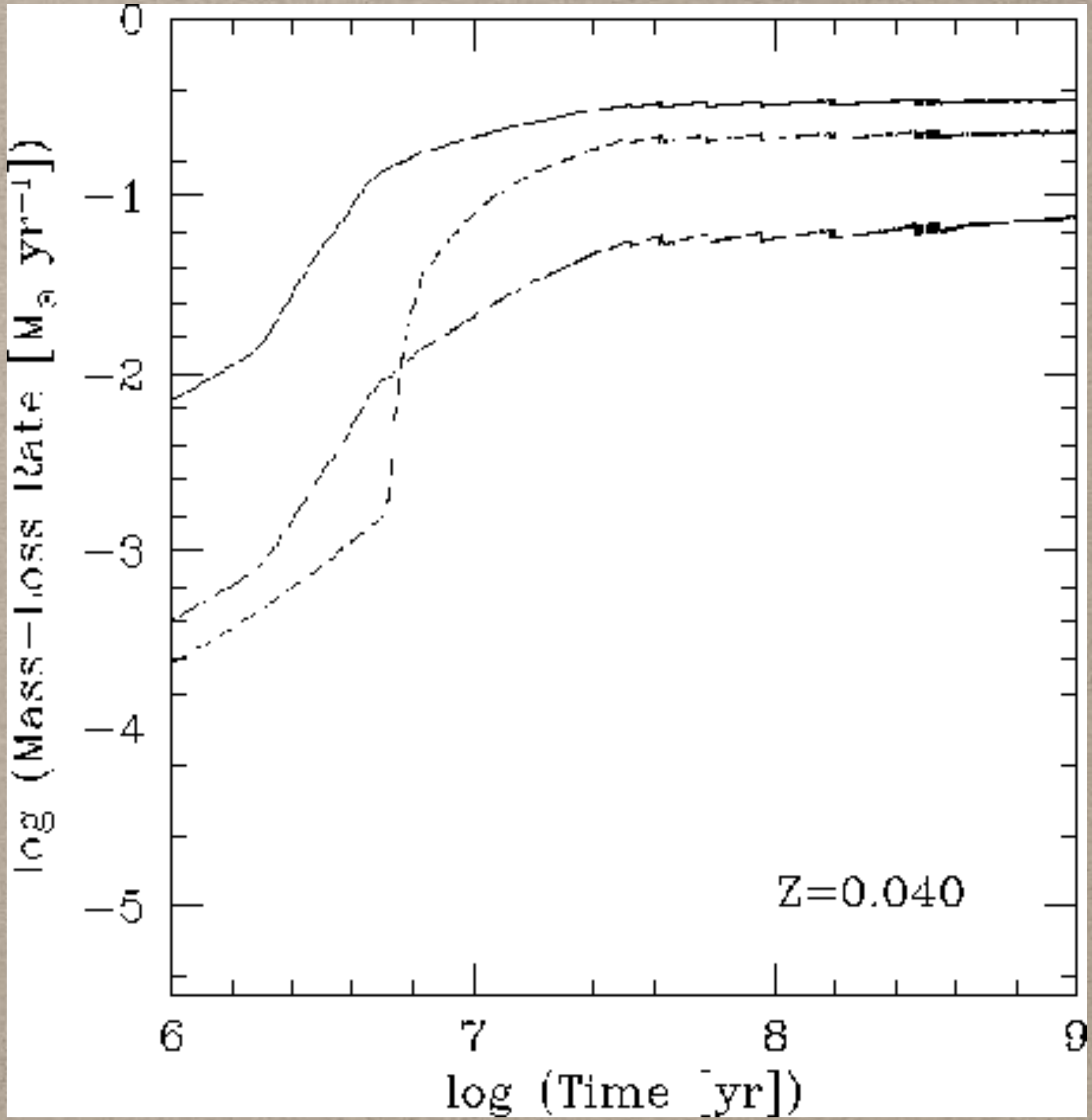




Exercise 1: Try to get such plots for Starburst99 website for both burst and continuous star formation

STELLAR MASS





$$M_*(t) = \int \dot{M}(SFR) - \dot{m}_{ej} dt$$

STAR FORMATION RATE FROM SPECTRAL MEASUREMENTS

Table 1.1. Model-based luminosity-to-SFR calibrations

Luminosity[a]	C^b	Assumptions[c]
L(UV)	$3.0 \times 10^{-47}\ \lambda$	0.1-100 $M_\odot$, $\tau \geq$ 100 Myr
L(UV)	$4.2 \times 10^{-47}\ \lambda$	0.1-30 $M_\odot$, $\tau \geq$ 100 Myr
L(UV)	$4.3 \times 10^{-47}\ \lambda$	0.1-100 $M_\odot$, τ = 10 Myr
L(UV)	$1.0 \times 10^{-46}\ \lambda$	0.1-100 $M_\odot$, τ = 2 Myr
L(TIR)	1.6×10^{-44}	0.1-100 $M_\odot$, τ = 10 Gyr
L(TIR)	2.8×10^{-44}	0.1-100 $M_\odot$, τ = 100 Myr
L(TIR)	4.1×10^{-44}	0.1-30 $M_\odot$, τ = 100 Myr
L(TIR)	3.7×10^{-44}	0.1-100& $M_\odot$, τ = 10 Myr
L(TIR)	8.3×10^{-44}	0.1-100 $M_\odot$, τ = 2 Myr
L(Hα)	5.5×10^{-42}	0.1-100 $M_\odot$, $\tau \geq$ 6 Myr, $T_e = 10^4$ k, n_e = 100 cm^{-3}
L(Hα)	3.1×10^{-41}	0.1-30 $M_\odot$, $\tau \geq$ 10 Myr, $T_e = 10^4$ k, n_e = 100 cm^{-3}
L(Brγ)	5.7×10^{-40}	0.1-100 $M_\odot$, $\tau \geq$ 6 Myr, $T_e = 10^4$ k, n_e = 100 cm^{-3}

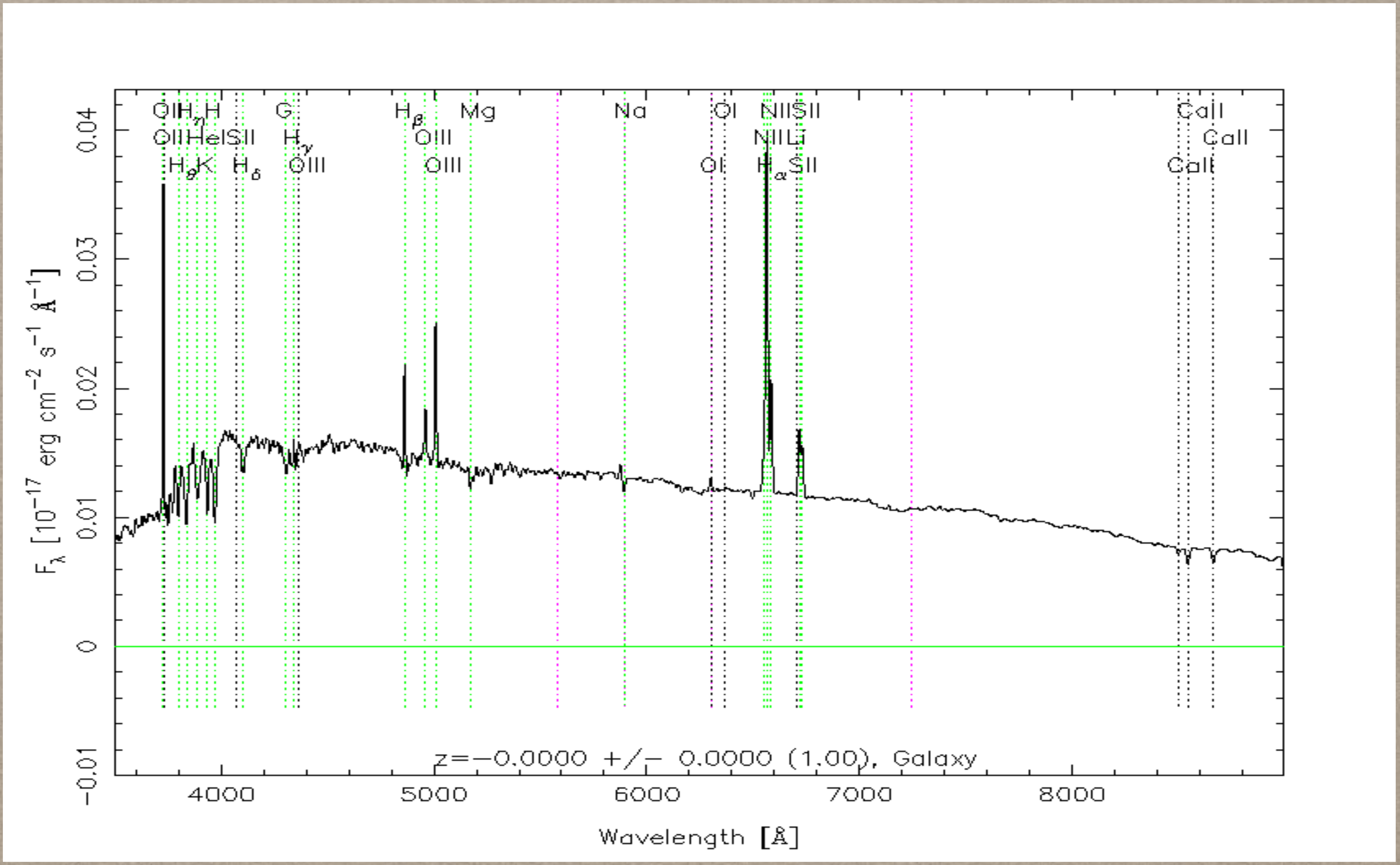
[a] Luminosity in erg s^{-1}. Stellar and dust continuum luminosities are given as $\nu L(\nu)$; total IR=TIR is assumed to be equal to the stellar population bolometric luminosity.

[b] The constant C appears in the calibration as: SFR(λ) = $C\,L(\lambda)$, where SFR is in units of $M_\odot$. The constant is derived from stellar population models, with constant star formation and solar metallicity (Starburst99, Leitherer et al. 1999). For SFR(UV), the numerical value is multiplied by the wavelength λ in Å.

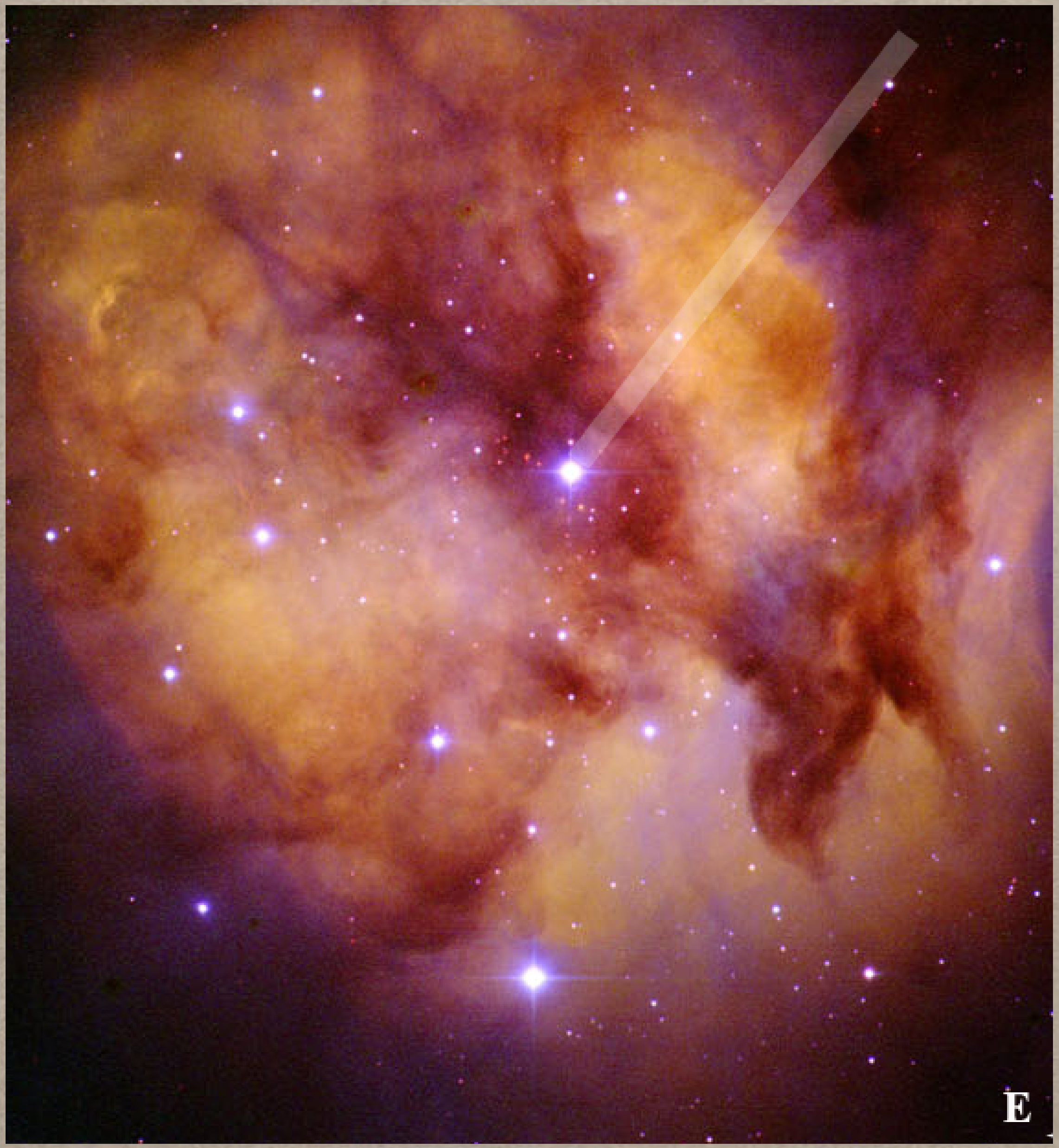
[c] Assumptions for mass range of the stellar IMF, which we adopt to have the expression derived by Kroupa (2001), see Section 1.2.2, and for the timescale τ over which star formation needs to remain constant, for the calibration constant to be applicable. For nebular lines, the adopted values of electron temperature and density are also listed.

https://ned.ipac.caltech.edu/level5/Sept12/Calzetti/Calzetti1_2.html

NEBULAR EMISSION



NEBULAR EMISSION



- Pure Hydrogen nebula

$n_{\rm HI}\Gamma_{\rm HI} = \alpha(T_e)n_e n_p$

$n_{\rm H} f_{\rm HI}\Gamma_{\rm HI} = n_{\rm H}^2(1 - f_{\rm HI})^2\alpha$

$\frac{f_{\rm HI}}{(1-f_{\rm HI})^2} = n_{\rm H}(\frac{\alpha(T_e)}{\Gamma_{\rm HI}})$

where $\Gamma_{HI} = \int \frac{4\pi J_\nu \sigma_{HI}(\nu) d\nu}{h\nu}$

is the photoionization rate of Hydrogen.

- Matter bound H II region.

$\dot{N} = \frac{4}{3}\pi R^3 n_e n_p \alpha(T)$

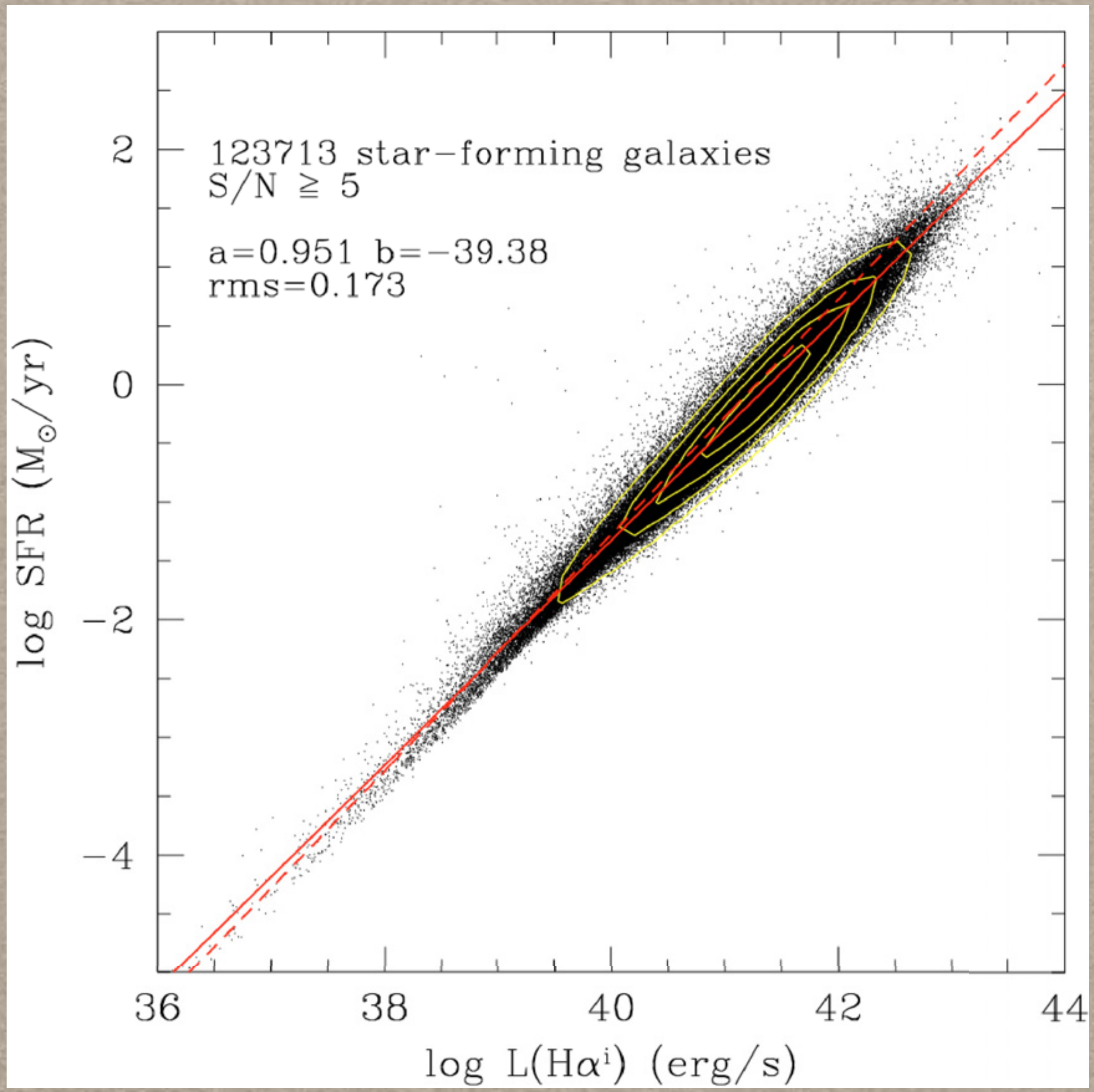
$R = \left(\frac{3\dot{N}}{4\pi n_{\rm H}^2 \alpha(T)}\right)^{1/3}$ Strömgren radius.

$L(Ly\alpha) = (2/3)\dot{N}h\nu$ and

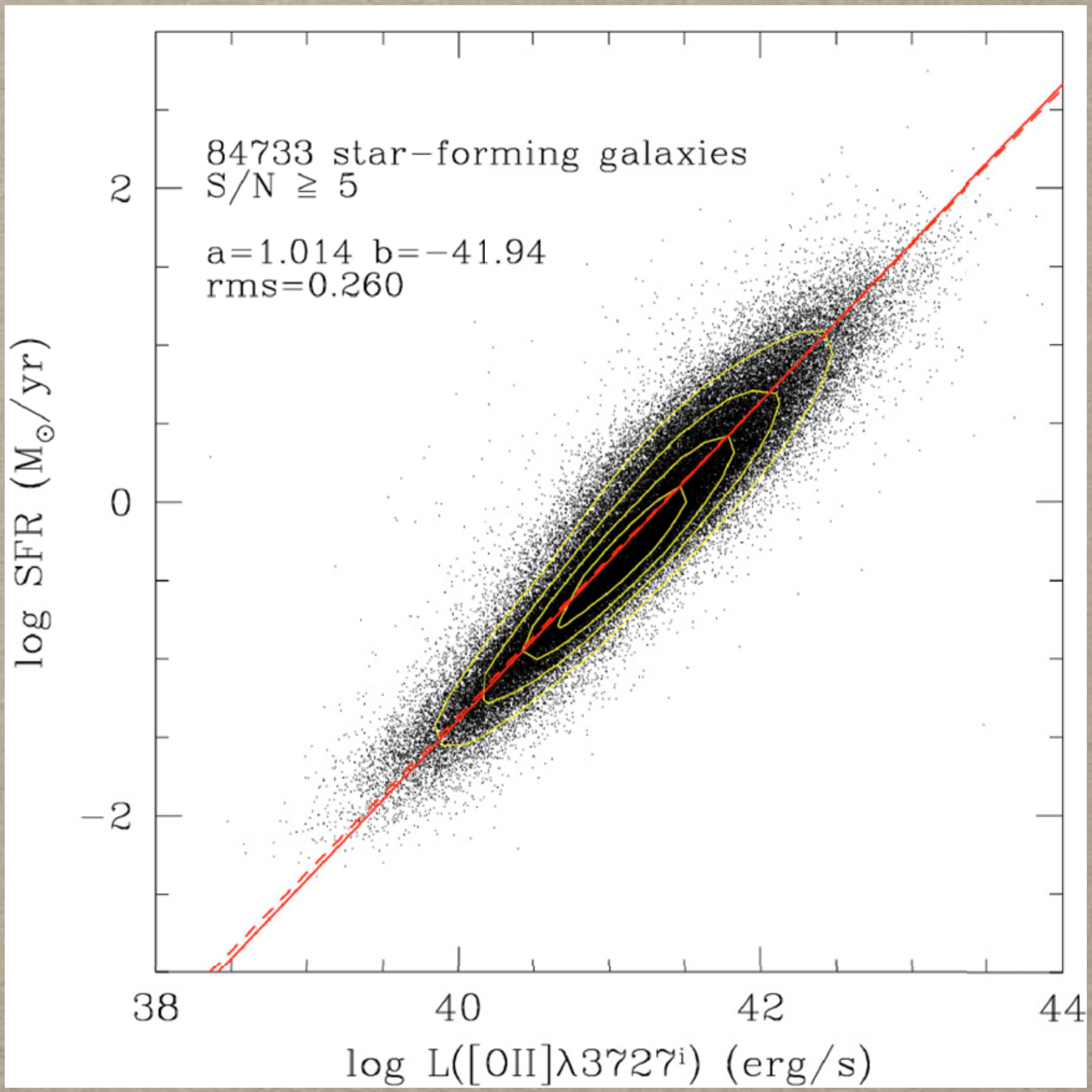
$L(H\alpha) = L(Ly\alpha)/8.7$

NEBULAR EMISSION & STARFORMATION

$$\log(\mathrm{SFR}) = \epsilon_{\mathrm{H}\alpha} \log L(\mathrm{H}\alpha) - \log(\eta_{\mathrm{H}\alpha}).$$



$$\begin{cases} \log \eta^{\mathrm{i}}_{\mathrm{H}\alpha} = 39.38 \pm 0.03 \\ \epsilon^{\mathrm{i}}_{\mathrm{H}\alpha} = 0.951 \pm 0.001 \end{cases}$$

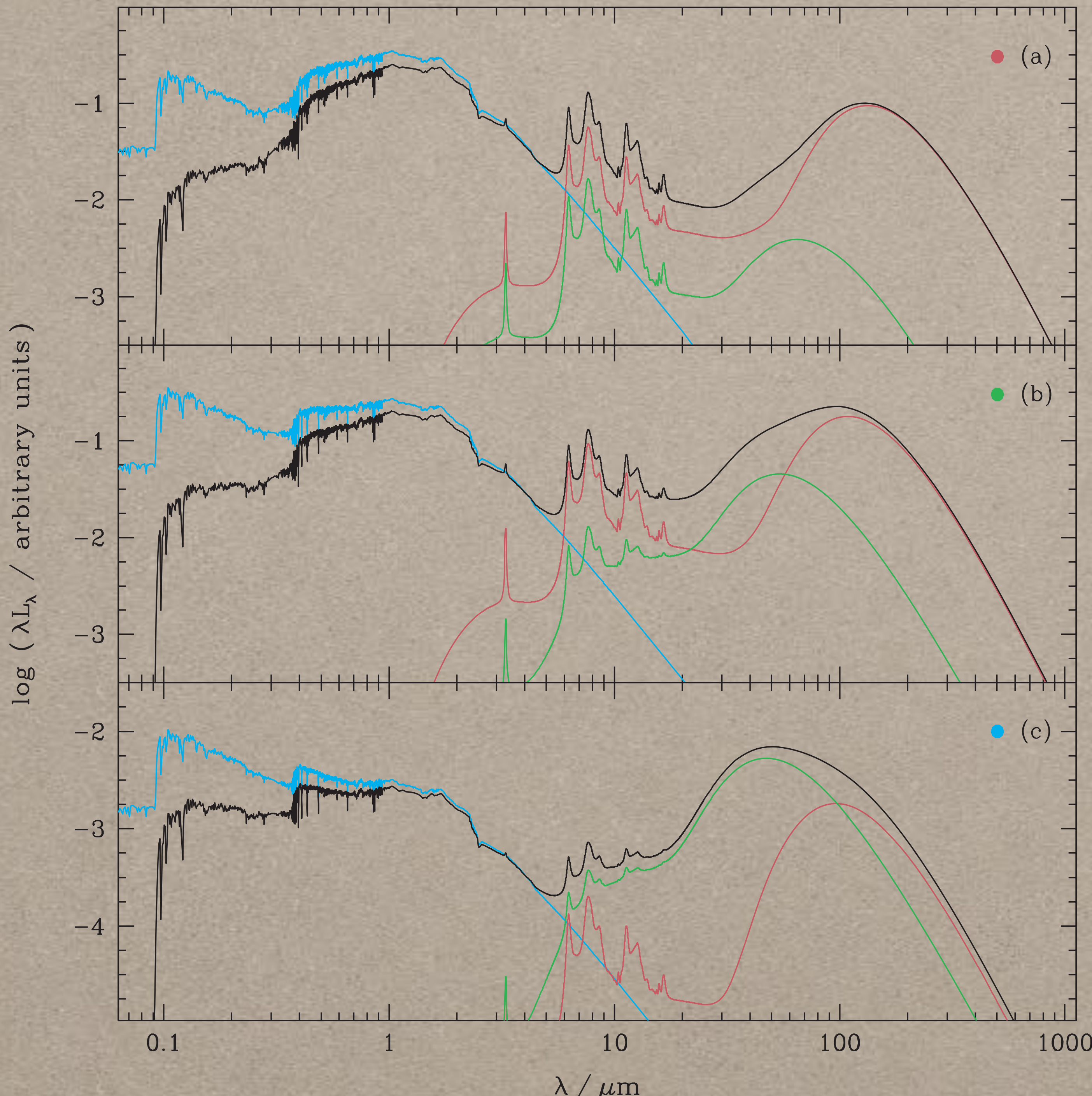


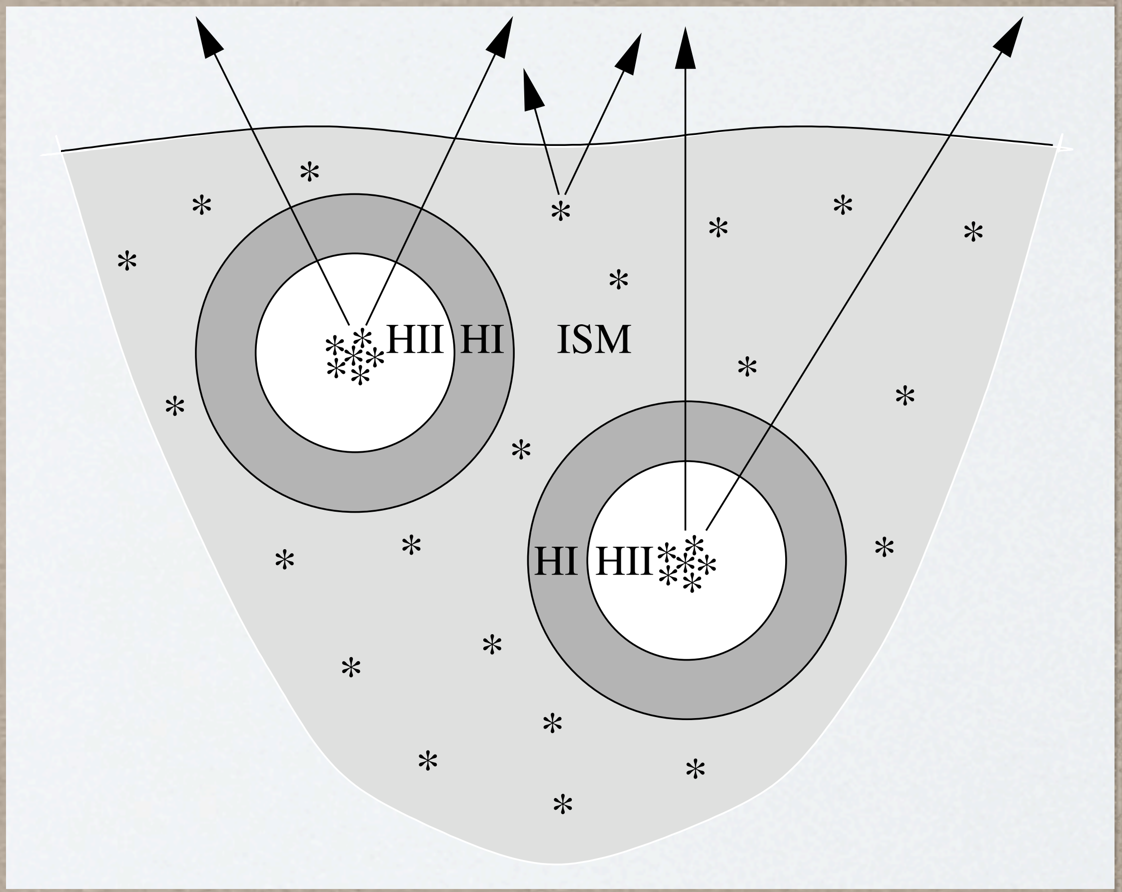
$$\begin{cases} \log \eta^{\mathrm{i}}_{[\mathrm{O\,II}]} = 41.94 \pm 0.7 \\ \epsilon^{\mathrm{i}}_{[\mathrm{O\,II}]} = 1.014 \pm 0.02 \end{cases}$$

$$\log \mathrm{SFR} = -39.21 + 0.982 \cdot \log L([\mathrm{O\,II}]) - 2.396 \cdot \log([\mathrm{O\,II}]/\mathrm{H}\beta) - 0.3354 \cdot \log([\mathrm{O\,II}]/\mathrm{H}\beta)^2 .$$

Agence &Lamareille, 2009, A&A, 495, 759

IR EMISSION AND STARFORMATION

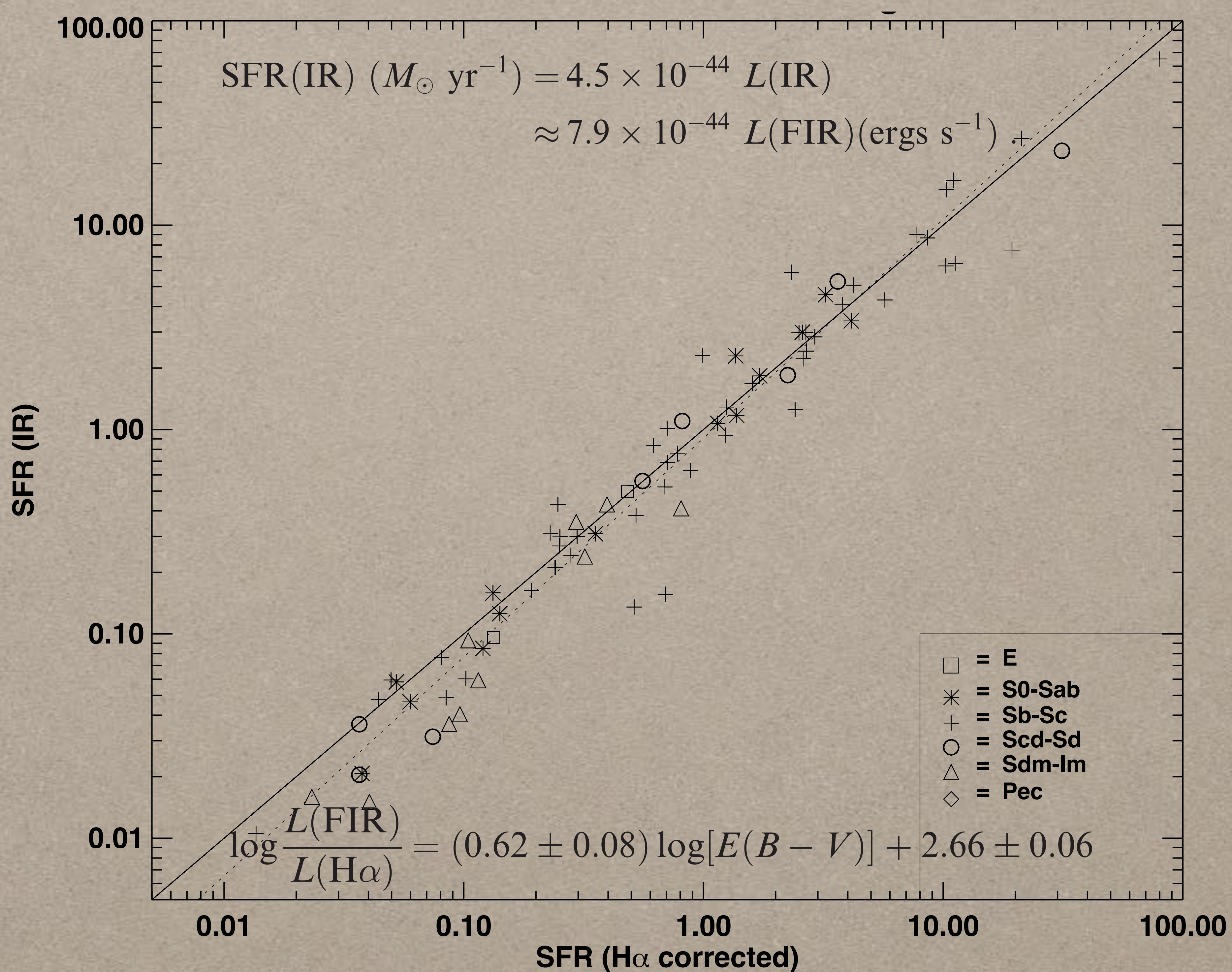




$SFR(FIR)(M_{\odot}yr^{-1}) = \frac{1}{\epsilon}4.5 \times 10^{-44} L_{FIR}(ergs^{-1})$
with
$L_{FIR} = 1.7 \times L_{60\mu m}$
ϵ is the fraction of UV/optical radiation absorbed by the dust.

Cunha et al., 2008, MNRAS, 388, 1595

IR EMISSION AND STARFORMATION



Kewley et al., 2002, ApJ, 124, 3135

NEBULAR EMISSION

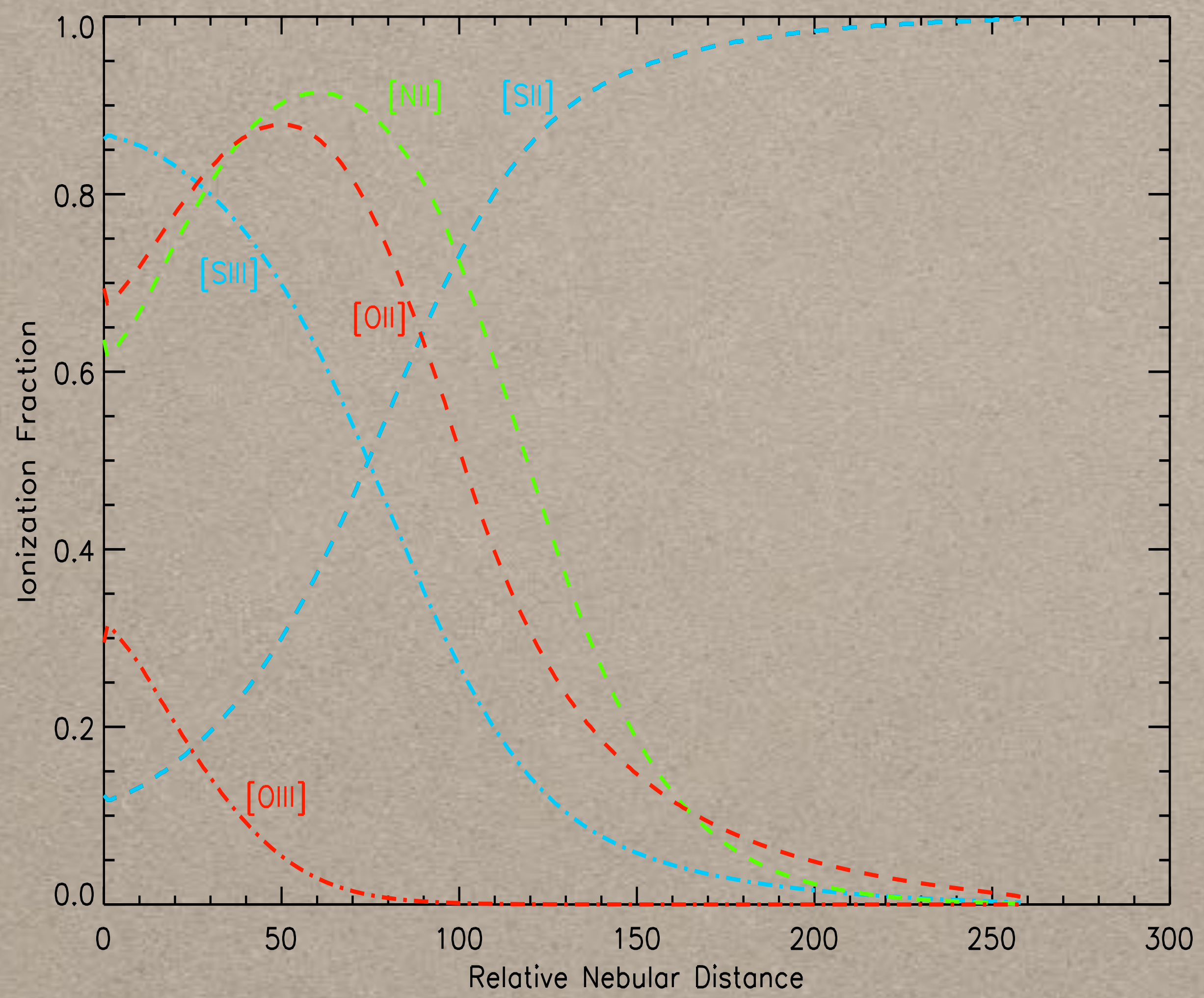


Figure 2. Relative ionization fractions for emission lines produced by Mappings III, plotted as a function of relative distance from inner surface of the nebula. The emission features shown here are [N II] (dashed green line), [O II] (dashed red line), [O III] (dash-dotted red line), [S II] (dashed blue line), and [S III] (dash-dotted blue line). For these outputs, a zero-age instantaneous burst SFH, a metallicity of $Z = 0.05\ Z_\odot$, an ionization parameter $q = 1 \times 10^7$, and an electron density $n_e = 100\ \mathrm{cm}^{-3}$ are assumed.

(A color version of this figure is available in the online journal.)

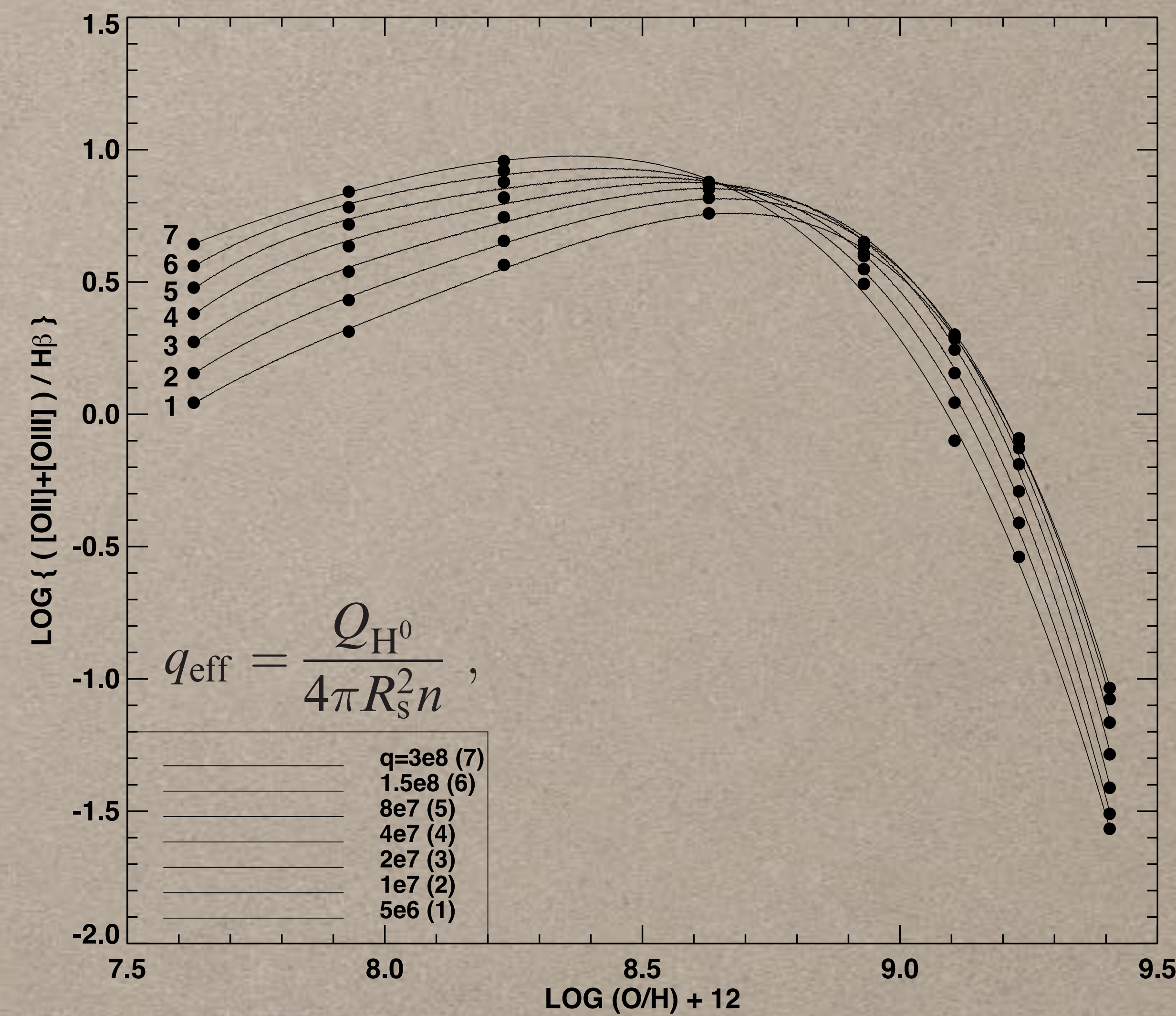
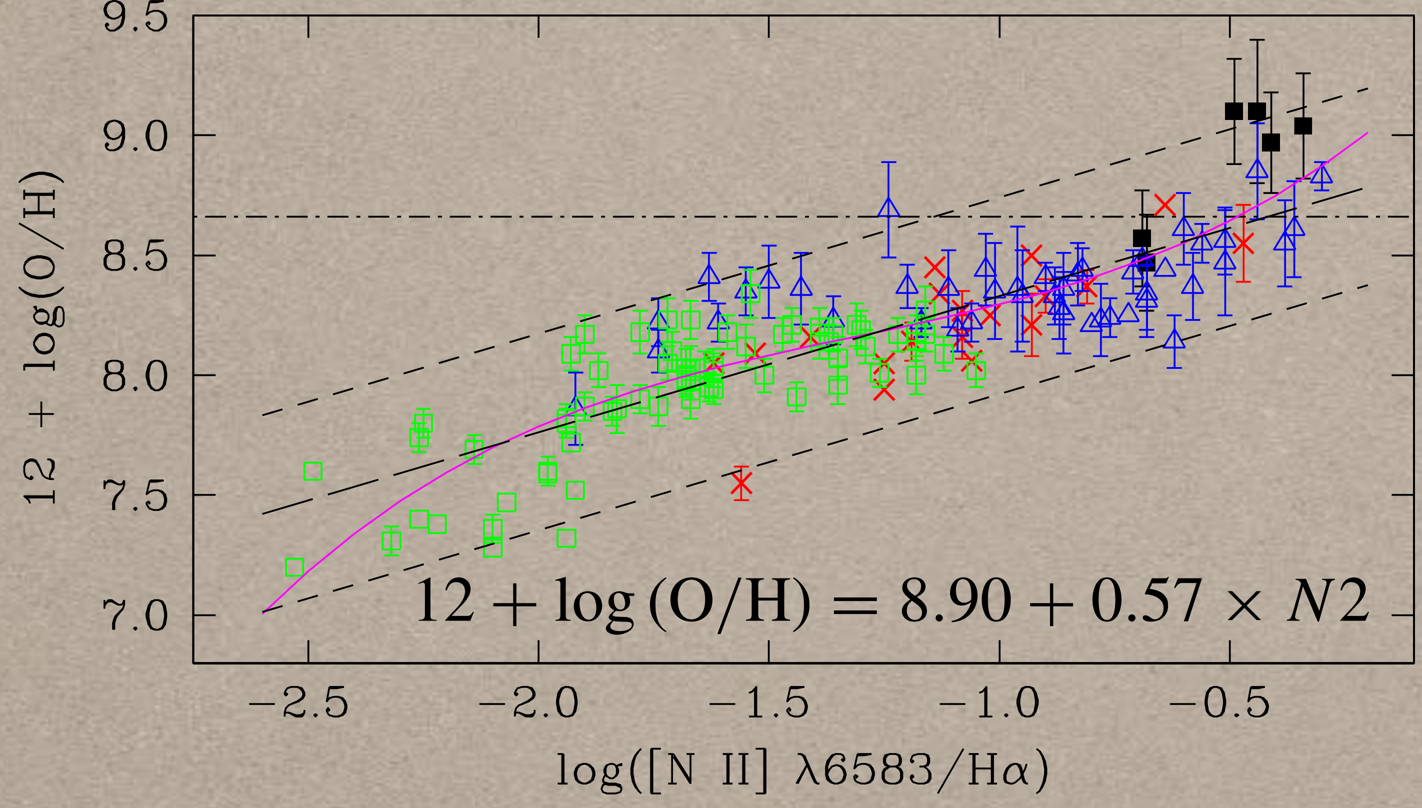
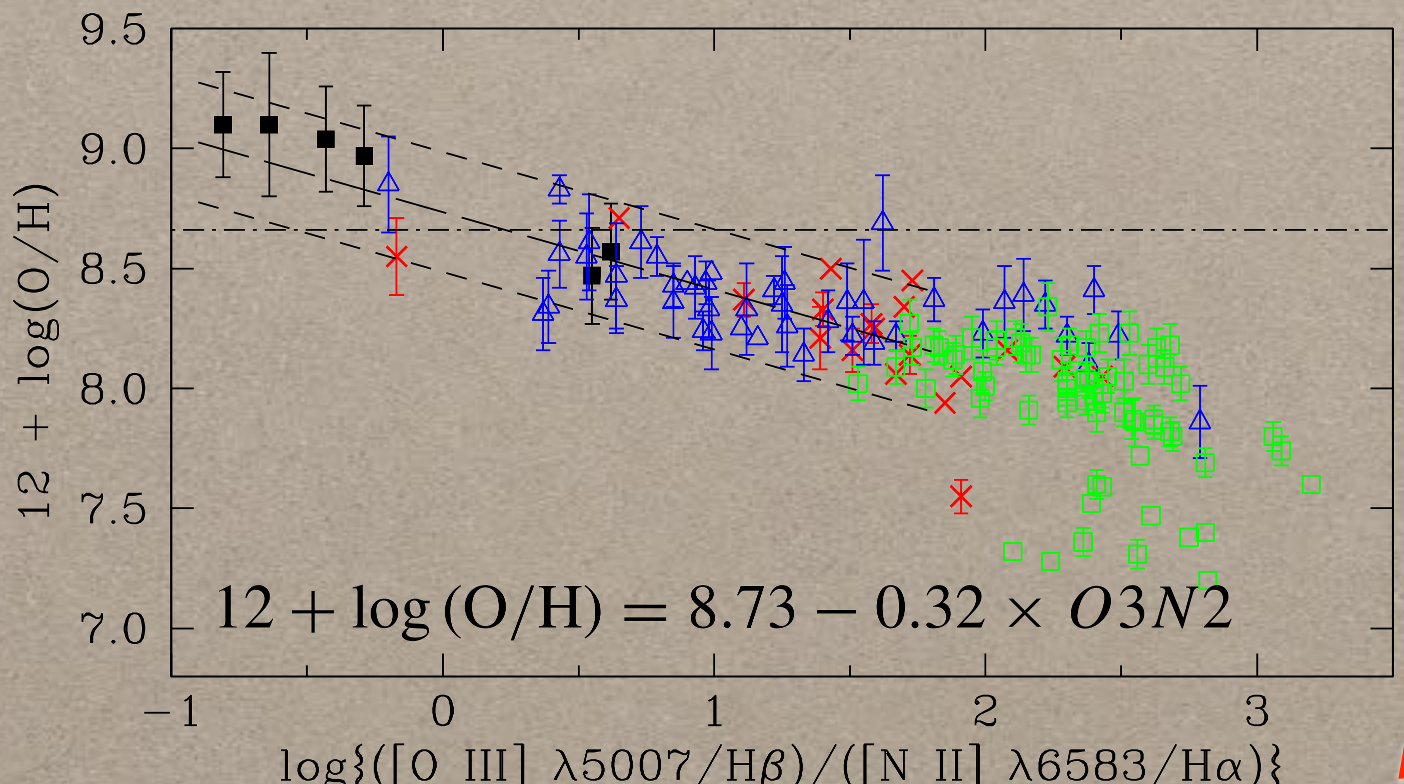


FIG. 5.—The log (([O II] λ3727 + [O III] $\lambda\lambda$4959, 5007)/Hβ) (R_{23}) diagnostic for abundance vs. metallicity. Curves for each ionization parameter between $q = 5 \times 10^6$ to 3×10^8 cm s^{-1} are shown. Filled circles represent the data points from our models at metallicities from left to right of 0.05, 0.1, 0.2, 0.5, 1.0, 1.5, 2.0, 3.0 $Z_\odot$. [*See the electronic edition of the Journal for a color version of this figure.*]

Levesque, 2010, AJ,139, 712; Kewley et al., 2002, ApJs, 142, 35

NEBULAR EMISSION & METALLICITY

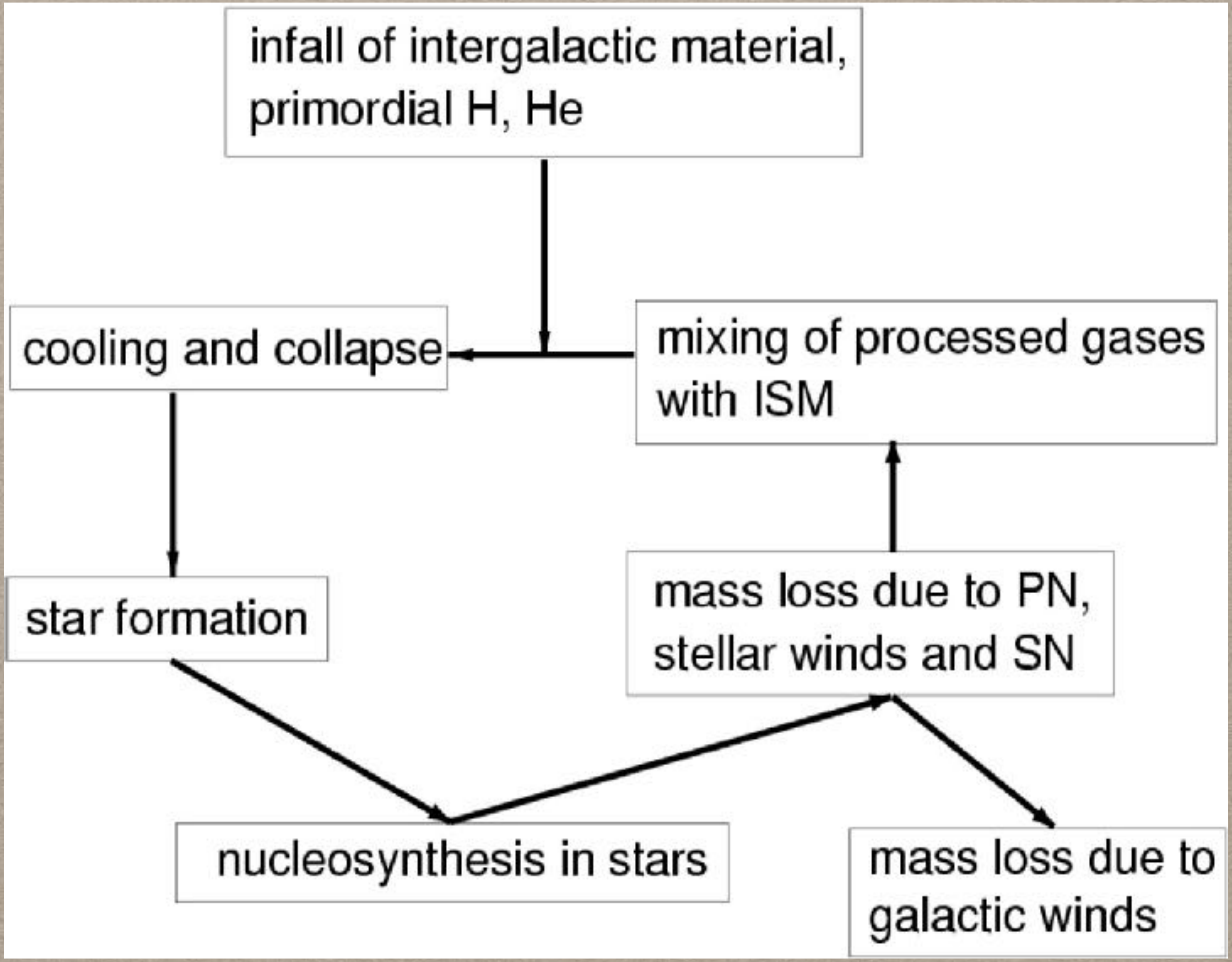




- Exercise 2: Measuring star formation rate, age, stellar mass, metallicity and dust content for a typical galaxy from the SDSS catalog. I will make a list of target and post it in SLACK for you.

Pettini & Pagel, 2004, MNRAS, 348, L59

GALAXY: METALLICITY EVOLUTION



From Bender's Lecture notes: Link in the last page.

GALAXY: METALLICITY EVOLUTION

- conservation laws:

(1) $M = M_s + M_g$	$\begin{cases} M & = \text{total mass} \\ M_s & = \text{mass in stars} \\ M_g & = \text{mass in gas} \end{cases}$
(2) $\frac{dM}{dt} = f - e$	$\begin{cases} f & = \text{rate of infalling gas} \\ e & = \text{rate of ejected gas} \end{cases}$
(3) $\frac{dM_s}{dt} = \Psi - E$	$\begin{cases} \Psi & = \text{star formation rate} \\ E & = \text{gas ejection rate of all stars} \end{cases}$
(4) $\frac{dM_g}{dt} = -\Psi + E + f - e$	

- Models with dM/dt = 0 are known as close box models.
- The star formation rate in a galaxy may be physically motivated for example ∝ available mass of cold atomic or molecular gas.

GALAXY: METALLICITY EVOLUTION

- gas ejection rate of all stars:

$$(5) \quad E(t) = \int_{m_t}^{\infty} [m - w_m] \Psi_{t-\tau(m)} \Phi(m) dm \qquad m = m_{ej} + \omega_m$$

m_t:	turnoff mass at time t = lowest mass of stars dying at time t
$m - w_m$:	ejected mass
$\Psi_{t-\tau(m)}\Phi(m)$:	birth rate at $t - \tau(m)$ = death rate at time t, stars of different generations are involved
$\tau(m)$:	main-sequence lifetime at mass m

Remnant mass
$$\begin{cases} w_m = 0.11m + 0.45M_\odot & (m < 6.8M_\odot) \\ w_m = 1.5M_\odot & (m \geq 6.8M_\odot) \end{cases}$$

N.B.: only assumption: mass loss at the end in negligible time
(wrong only for $\Delta t < 10^6$ because of O-star winds)

GALAXY: METALLICITY EVOLUTION

- evolution of the abundance Z (relative mass fraction) for one or more metals:

$$(6) \quad \frac{d(ZM_g)}{dt} = -Z\Psi + E_Z + Z_f \cdot f - Ze$$

E_Z: ejection rate of metals from stars, SNe etc.
$Z_f \cdot f$: infalling metals per time
ZM_g: mass of metals in the gas

$M_m = M_g Z$

Z_e = metal ejection rate into the IGM

- ejection rate of metals

$$(7) \quad E_Z(t) = \int_{m_t}^{\infty} [(m - w_m) Z_{t-\tau(m)} + m p_{Zm}] \Psi_{t-\tau(m)} \Phi(m) dm$$

$(m - w_m) Z_{t-\tau(m)}$: mass of metals that at time $t - \tau(m)$ were locked in a star of mass m and are now ejected with the envelope at time t; often also written as $(m - w_m - m p_{Zm}) Z_{t-\tau(m)}$ (see Tinsley 1980)

$m p_{Zm}$: new metals produced by a star of mass m which originally formed from gas with metallicity Z
<u>Note</u>: for some elements (e.g. Li) $p_{Zm} < 0$

GALAXY: METALLICITY EVOLUTION

- returned mass per mass of stars formed

(8) $$R = \int_{m_1}^{\infty} (m - w_m)\Phi(m)dm$$

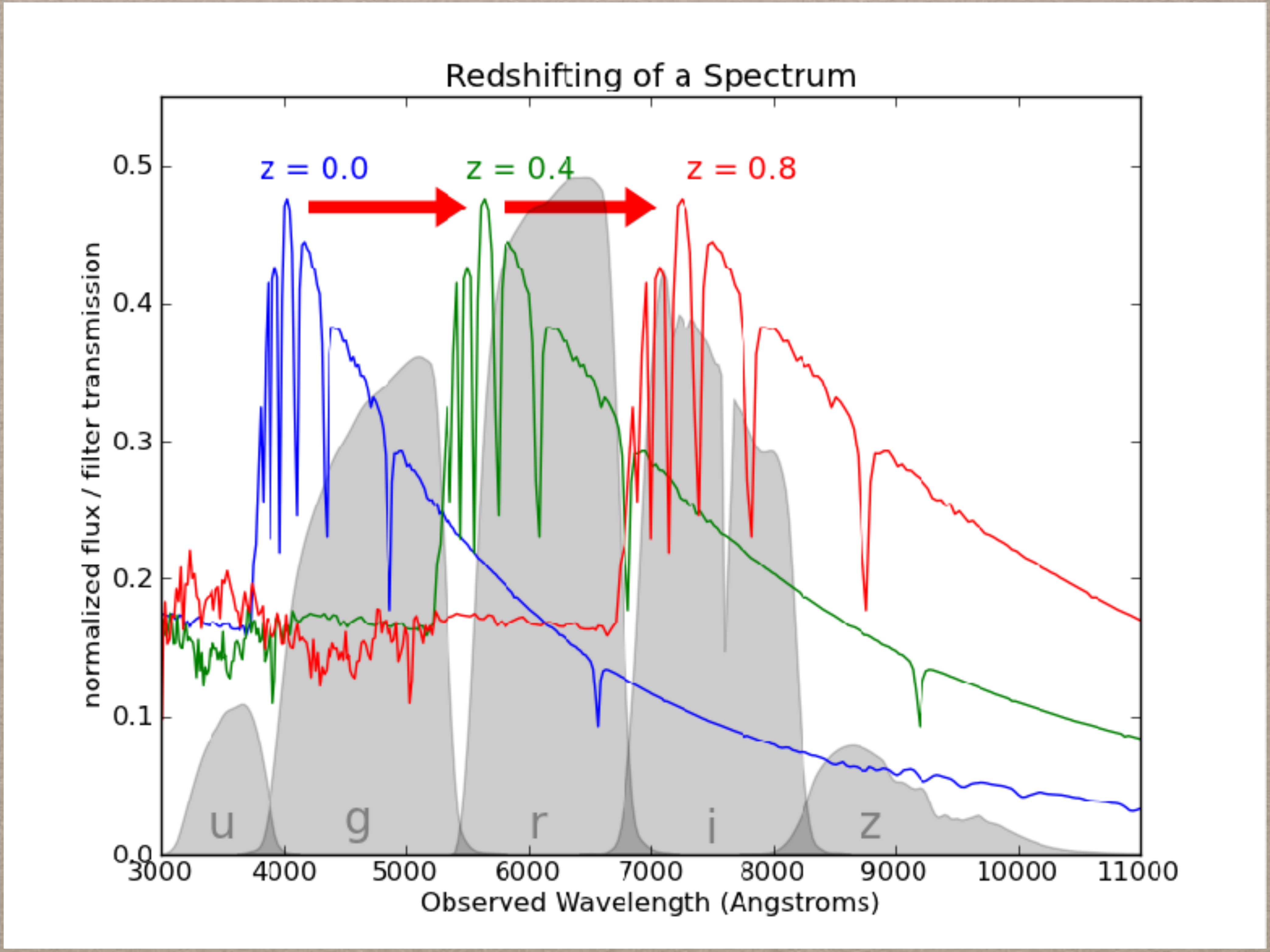
equation (8) is independent of Ψ and is valid for one (!) generation of stars.

- Yield (mass of produced metals per remnant mass)

(9) $$y = \frac{1}{1-R}\int_{m_1}^{\infty} m p_{Zm}\Phi(m)dm$$

yield y: mass of newly produced metals by a populations of stars which after their deaths have a sum of remnant masses of $1M_\odot$.

K-CORRECTION



K-CORRECTION

Basic set of equations

Hogg et al., astroph/0210394

$m_R = M_Q + DM + K_{QR}$ where

$DM = 5\ log_{10}[\frac{\mathrm{D_L}}{\mathrm{10pc}}]$; the distance modulus

$$m_R = -2.5\ log_{10}\left[\frac{\int \frac{d\nu_0}{\nu_0} f_\nu(\nu_0) R(\nu_0)}{\int \frac{d\nu_0}{\nu_0} g_\nu^R(\nu_0) R(\nu_0)}\right]$$

$$M_Q = -2.5\ log_{10}\left[\frac{\int \frac{d\nu_e}{\nu_e} \frac{L_\nu(\nu_e)}{4\pi(10pc)^2} Q(\nu_e)}{\int \frac{d\nu_e}{\nu_e} g_\nu^Q(\nu_0) Q(\nu_0)}\right]$$

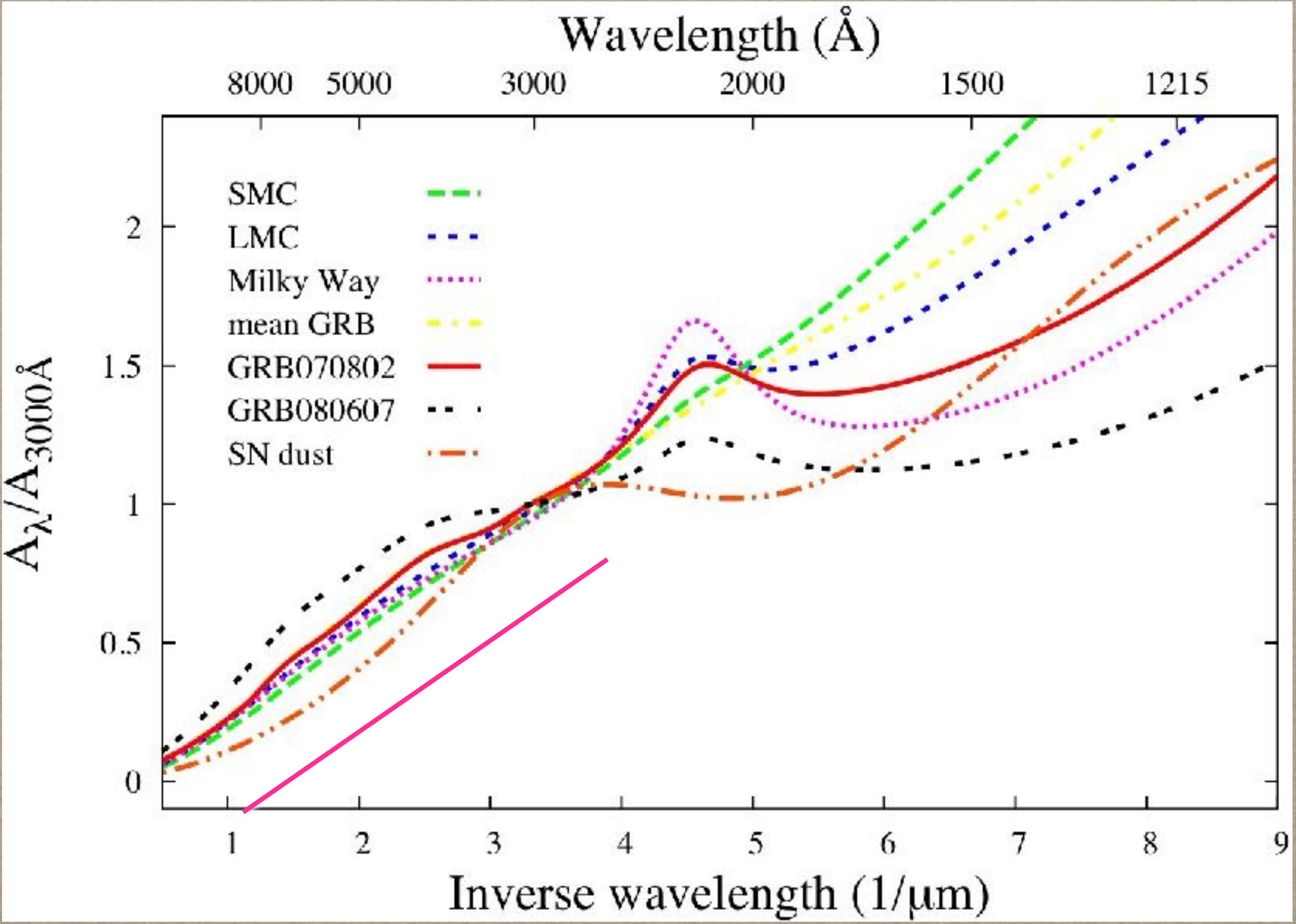
$L_\nu(\nu_e) = \frac{4\pi D_L^2 f_\nu(\nu_0)}{(1+z)}$ and $\nu_e = [1+z]\ \nu_0$

$$K_{QR} = -2.5\ log_{10}\left[[1+z]\frac{\int \frac{d\nu_0}{\nu_0} L_\nu([1+z]\nu_0) R(\nu_0) \int \frac{d\nu_e}{\nu_e} g_\nu^Q(\nu_e) Q(\nu_e)}{\int \frac{d\nu_0}{\nu_0} g_\nu^R(\nu_0) R(\nu_0) \int \frac{d\nu_e}{\nu_e} L_\nu(\nu_e) Q(\nu_e)}\right]$$

K-CORRECTION

- Remember there is no K-correction for bolometric magnitudes and integrated line fluxes.
- As the galaxy spectral energy distribution depends on IMF, star formation rate history, metallicity evolution and binary fraction etc for a given object there is no unique solution. Only way to move forward is to fit SED over a wider spectral regions.
- An useful reference is: Blanton & Sam, 2007, ApJ, 133, 734

EXTINCTION CURVES



EXTINCTION CORRECTIONS

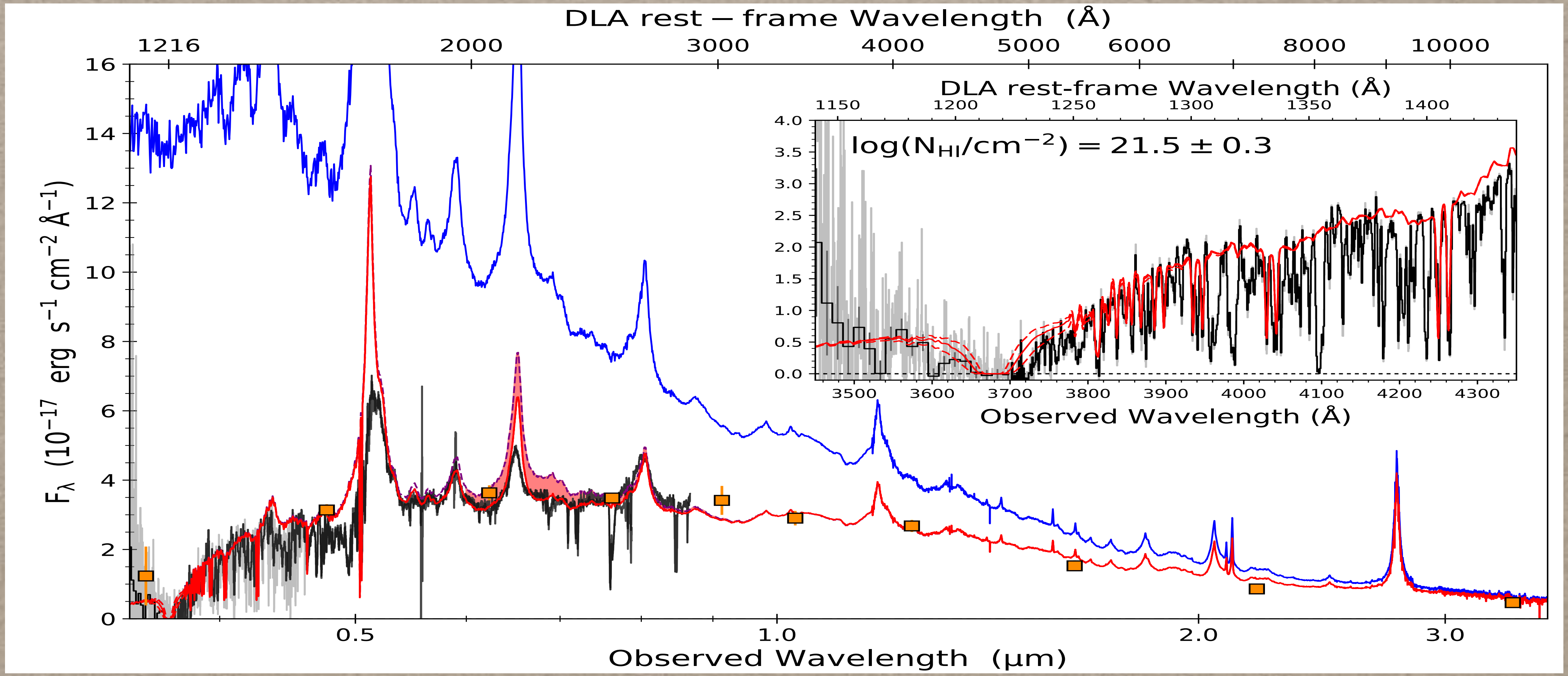
$$A_\lambda = m_\lambda^0 - m_\lambda$$

$$A_V = R_V \times E(B-V)$$

$$A_V = 1.086 \times \tau_V$$

$$\frac{N_H}{A_V} = 1.8 \times 10^{21} cm^{-2} mag^{-1}$$

$$\lambda_o = \lambda_e(1+z_e) = \lambda_d(1+z_d)$$



EXTINCTION: BALMER DECREMENT

$$F_{H\alpha} = F^0_{H\alpha} exp(-\tau_{6563})$$

$$F_{H\beta} = F^0_{H\beta} exp(-\tau_{4863})$$

$$\frac{F_{H\alpha}}{F_{H\beta}} = \frac{F^0_{H\alpha}}{F^0_{H\beta}} exp(-\tau_{6563} + \tau_{4863})$$

where $\frac{F^0_{H\alpha}}{F^0_{H\beta}} = 2.86$ for case-B recombination at $T = 10^4$ K.

τ_λ comes from an assumed extinction curve and E(B-V) values.

$$f^{\mathrm{i}}_\lambda(\lambda) = f_\lambda(\lambda) \cdot \exp\left(\frac{\tau_V}{\tau(V)} \cdot \tau(\lambda)\right).$$

$$\tau_V^{\mathrm{Balmer}} = \frac{\ln(\mathrm{H}\alpha/\mathrm{H}\beta) - \ln(\mathrm{H}\alpha^{\mathrm{i}}/\mathrm{H}\beta^{\mathrm{i}})}{\tau(\beta) - \tau(\alpha)} \cdot \tau(V).$$

USEFUL REFERENCES:

- http://www.astro.bas.bg/~petrov/bender00.html
- http://www.stsci.edu/science/starburst99/docs/default.htm
- http://adsabs.harvard.edu/abs/2003MNRAS.344.1000B
- http://kcorrect.org/